Paris Meeting, Aug. 7th, 1900.

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TRANSACTIONS

OF THE

AMERICAN INSTITUTE

OF

ELECTRICAL ENGINEERS.

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AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

New York, February 28th, 1900.

The 140th meeting of the INSTITUTE was held this date at No. 12 West 31st Street and was called to order by President Kennelly, at 8.15 P. M.

The Secretary announced the election at the meeting of the Executive Committee in the afternoon of the following associate members:

BARNES, HOWELL HENRY	General Engineer, Mexican Electric Works Ltd. Apartado, 905 Mexico City.	C. F. Beames. H. S. Wilson. J. W. Thompson.
BOYD, JOHN DUNCAN	Electrician, Yuba Electric Power Co., Marysville, Cala.	F. V. T. Lee. T. E. Theberath. F. A. C. Perrine.
EVANS, CLEMENT W.	Electrical Engineer, American Engineering Co., Box 2100 Mexico City.	C. F. Beames. J. W. Thompson. H. S. Wilson.
HENRY, LEWIS WARNER	Assistant in Engineering Department, Mexican General Electric Co., Mexico City.	C. F. Beames. A. E. Worswick. H. S. Wilson.
HUNT, A. M.	Consulting Engineer, 331 Pine Street, San Francisco, Cala.	Clarence L. Cory. R. S. Masson. Wynn Meredith.
LAWRENCE, WM. G.	Manager of Light and Power Department, Town of Hudson, Hudson, Mass.	C. B. Burleigh. Geo. P. Low. Wynn Meredith.
MCCREARY, J. L.	Constructing Engineer, District Railway Co., Mexico City.	A. E. Worswick. C. F. Beames. J. W. Thompson.
MCVAY, H. D.	City Electrician, City of Wichita, Wichita, Kansas.	Samuel Sheldon. Herbert L. Webb. C. O. Mailloux.
NEURATH, MORRIS M.	Consulting Engineer, 1444 Monadnock Block, Chicago, Ill.	Geo M. Mayer. S. G. Neiler. R. H. Pierce.

RENSTROM, FRANS OSCAR	Superintendent, The Regla Power Co., Apartado 95, Pachuca, Mexico.	C. F. Beames. J. W. Thompson. H. S. Wilson.
SCHIAFFINO, MARIANO L.	Chief Electrician, Compania de Luz Electrico, Guadalajara, Mexico.	C. F. Beames. J. W. Thompson. H. S. Wilson.
SMITH, WALTER EUGENE	Electrician, The United Electric Improvement Co., 19th and Allegheny Ave.; residence. 2010 Ontario Street, Philadelphia, Pa.	R. H. Klauder. Herbert Lloyd. J. B. Entz.
WILEY, GEO. LOURIE	Manager, Standard Underground Cable Co., 18 Cortlandt St., New York; residence, Arlington, N. J.	John J. Carty. Ralph W. Pope. T. D. Lockwood.
ZAPATA, J. M.	Constructing Engineer, The Mexican General Electric Co., Apartado 403, Mexico City.	C. F. Beames. J. W. Thompson. H. S. Wilson.

Total, 14.

Transferred from associate to full membership.

Approved by Board of Examiners, January 12th, 1900.

EDWARD J. WILLIS	Steam and Electrical Engineer, Richmond, Va.

Dr. Pupin read the following note.

A note presented at the 190th Meeting of the American Institute of Electrical Engineers, New York, February 28th, 1900. President Kennelly in the Chair.

A FARADMETER.

BY M. I. PUPIN.

The art of measuring the capacity of a condenser has not yet reached that stage of perfection which can be justly claimed for the resistance measurement. The prevailing method is the ballistic galvanometer method. Leakage and absorption can and often do introduce serious errors into this method. These errors can be reduced to any desirable limit by employing alternating currents of appropriate frequency.

This was the principal consideration which led to the construction of the faradmeter which forms the subject of this note. In addition to this consideration, which concerns the accuracy, there is another very important consideration which must be taken into account and that is the convenience of the method and the cheapness and durability of the apparatus to be employed. Much of the capacity measurement work, especially in connection with telegraphy, telephony, and the construction of alternating current machinery, has to be done under conditions under which it is quite inconvenient to employ the ballistic galvanometer. I believe that the faradmeter described here will be found to answer quite satisfactorily all reasonable requirements as regards convenience.

The theory on which the construction of this meter is based is very simple, as follows: Let $x\ y$ (Fig. 1) be a conductor through which an alternating current flows. D is a condenser of known capacity and E is a condenser of unknown capacity; F is a differentially wound telephone. Connect as indicated and adjust the resistances A B and B C until silence is obtained in the

telephone. Then capacity of D : capacity of E :: resistance of B C : resistance of A B. It is understood, of course, that the

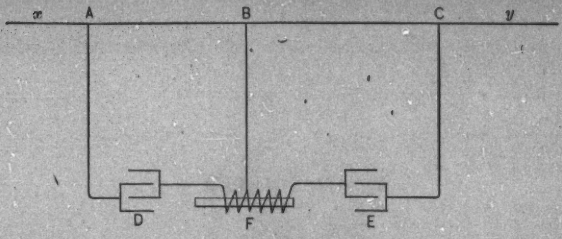


Fig. 1.

resistances A B and B C are non-self-inductive and that the capacity reactance of each condenser is in its own circuit by far the greatest element of the impedance. It will be seen presently that these conditions are fulfilled in the apparatus shown in Fig. 2 which our fellow member, the well-known mechanician, Mr. Baillard, constructed for me and employed in some work which he has been doing for me during the last two months.

In Fig. 2, A is a cell which feeds into the primary B of a small induction coil provided with an interrupter such as is used in

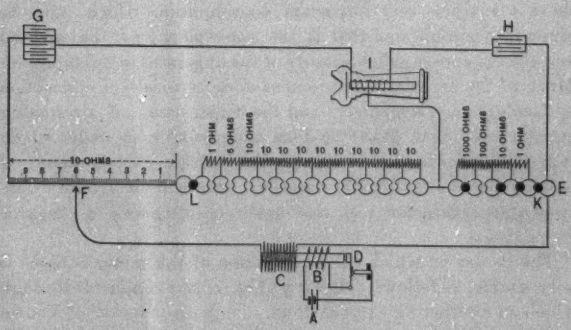


Fig 2.

electrotherapy. The secondary C is connected to a row of resistance coils E and L. These resistances are arranged in conven-

ient steps as indicated. The resistance coils L have in addition several equal lengths of manganin wire F stretched over a graduated scale. The resistance of the coils as well as of the manganin wire is carefully calibrated once for all. The condenser H is a carefully constructed mica condenser of known capacity. G is the condenser the capacity of which is to be determined. I is the differentially wound telephone. The approximate adjustment is accomplished by the plugs which control the resistances E and L, and the final adjustment is made by the manganin wire by a sliding contact F which varies the length of the manganin wire to be introduced, and therefore varies the drop which acts on one of the condensers. With ordinary frequencies, capacities up to several microfarads can be determined with an accuracy of a small fraction of one per cent. easily and rapidly.

It is self-evident, of course, that in place of the differentially wound telephone I, we can use an ordinary telephone and place it in the bridge which connects a point between G and H to a point between the resistance coils E and L. The conditions of balance are the same as in the method employing the differentially wound telephone. In many practical cases this second method is preferable.

Mr. Baillard has experimented quite a great deal with a faradmeter of this kind and is better prepared to go into details of the apparatus and the method than I am.

Discussion.

The President:—The apparatus which is before you represents a means for measuring capacity as a shop instrument, or as a swiftly measuring engineering instrument, contrasted with the older method of galvanometer work, and introduces the peculiar idea that the capacity of a condenser, as measured in accordance with its commercial use in alternating current phenomena, may be somewhat different from its capacity as measured according to the ordinary definition and in the ordinary way. In other words the capacity, as defined by the charge per unit of pressure applied at the terminals, is not exactly the same in every practical case, as the capacity of a condenser determined by its impedance to the flow of alternating current, and in this case the impedance method is represented, which is very swift, and no doubt supplies a very great want in workshop practice. Of course the same plan of measuring capacity by means of alternating currents is not entirely new, but the apparatus which is before us possesses very interesting features.

Mr. Carl Hering:—I would like to ask Dr. Pupin whether the result obtained by such a measurement of a capacity would be likely to be different from that obtained by the usual method, and if so about how much might it differ.

Dr. Pupin:—I know that there is a difference, and I never had much confidence in capacity measurements of paraffin condensers. With the ballistic galvanometer method the differences will depend. of course, on the experimenter. It may be large, or it may be small, depending on what precautions you take. The other day Mr. Baillard and myself took a condenser made by a manufacturer whose name I do not care to mention. It was sold for a one-half microfarad standard; we used it as such. Mr. Baillard made a lot of condensers for me and measured their capacity in terms of this standard. We began to suspect the standard, and I put in a slight protest against it. Then we determined it in terms of another standard by the method described this evening, and found it forty per cent. off. But if its capacity was determined by the ballistic galvanometer, it could be made to read all right. Now the reason for that was, we found afterwards, that it had too much leakage. The insulation resistance was only 250,000 ohms. Well, of course if we have an insulation such as that, and determine the capacity by the ballistic galvanometer method, we may get anything for capacity. I know it as a matter of fact that paraffin condensers may have considerably smaller capacity when determined by alternating currents than the capacity obtained by the ballistic galvanometer; but I am not prepared to say exactly how much. But I certainly am prepared to say that in many cases it is more than good engineering practice ought to permit. If the paraffin condenser is made in the most careful way, under a vacuum,

heated, and so forth, so that it contains no air and no moisture, and other imperfections, then it would be all right. But if a paraffin condenser is made in the old-fashioned way, without any modern precautions being taken while being manufactured, then the difference can be certainly over five per cent., and that is more than good engineering practice ought to permit. I do not think that these precautions have to be observed as rigidly in the manufacture of mica condensers, or condensers made of hard rubber, where the insulation is very high, and the specific inductive capacity does not vary much with voltage; but in paraffin paper it makes quite a difference.

DR. LOUIS DUNCAN:—May I ask whether the question of capacity—ten per cent.—is not a question between absorption and the treatment of condensers. You may make a paraffin condenser, and the question about what its capacity is, is a question of absorption largely, also of manufacture of the condenser. It might vary not only five per cent. but 25.

DR. PUPIN:—It may be anything at all, where there is much leakage current.

DR. DUNCAN:—So that capacity is not so important as the history of the condenser?

DR. PUPIN:—The history of the condenser exactly. It may be anything. You cannot tell what it is going to be, voltage leakage, etc., have to be taken into consideration.

MR. TOWNSEND WOLCOTT:—I would like to ask if this current used was simply an interrupted current.

DR. PUPIN:—An alternating secondary, produced by an interrupted primary current. Where there is an alternating current it would be very much better to dispense with the buzzer of the faradmeter and use the alternating current. The other day I was down at the New York Telephone Building, and I inquired about the capacity of the cables, in which something like vulcanized rubber was used for insulation, and I was told it was so and so much per mile. I said: Are you sure about that? They said: We are not sure at all. We determined it by ballistic galvanometer, and we do not know how near we are to the true capacity. Although methods of the class to which the method described this evening belongs are not, I suppose, altogether new—I do not think they are; they are too self-evident to be new—but the fact is that nobody has reduced them to practice.

MR. CHARLES S. BRADLEY:—Do you think that a cable would take its charge quick enough to be treated the same as a condenser?

DR. PUPIN:—Well, the cable should not be allowed to take its charge in the ordinary commercial sense—that is to say, the cable should not be given time enough to soak in its charge.

MR. BRADLEY:—But if it is a long cable, it will take some time for the charge to reach the other end.

DR PUPIN:—Oh, yes. You mean the distributed capacity. Thirty periods mean six thousand miles wave length, very

roughly. Up to 300 or 400 periods per second there would be no trouble. You could take a cable of 300 or 400 miles length and determine its distributed capacity just as if it were an ordinary condenser.

MR. CARL HERING:—It seems to me this method is especially applicable for measuring a capacity under the very conditions under which it is to be used in practice, and it therefore has some advantages over the ballistic method. The frequencies of telegraph, telephone or power currents are quite different from each other. I should imagine that with this method different results would be obtained with different frequencies, and in measuring the capacities of cables or any other apparatus, the frequency of the current used for the test might be made equal to that with which the apparatus is to be used in practice, as, for instance, those for telegraphic, telephonic or power purposes. The virtual or effective capacity for that particular frequency might therefore be determined more reliably in this way than by the usual ballistic method.

DR. PUPIN:—Different frequencies and different potentials—the capacity of the paraffin condenser depends to some extent on the difference of potential applied to it.

DR. DUNCAN:—Could you use different voltage in this method?

DR. PUPIN:—Yes, you can.

MR. DUNCAN:—Provided the resistance will stand it.

DR. PUPIN:—Oh, yes. Well, that is simply a question of spending money to get the proper resistances.

DR. DUNCAN:—It would not affect anybody here.

DR. PUPIN: The companies interested in capacity measurements have money enough to spend for things of this kind.

MR. H. D. REED:—As I understand Dr. Pupin, a ballistic galvanometer has given satisfaction where the insulation was high and steady. Well, you take a cable, any that a telephone company would have, would not be over a mile in length; most of them are much shorter. Now, the resistance of those rubber cables, as a rule, will run from 1,000 to 2,000 megohms per mile. In a short length of cable where your resistance was several times that, I do not see why the ballistic galvanometer would not be all right.

DR. PUPIN:—I did not say insulation alone. Absorption has to do with it, too. You have got to take that into consideration.

THE PRESIDENT:—If there is no further discussion of this paper, we will pass to the next, which is entitled "Notes on Electric Traction Under Steam Railway Conditions," by Mr. Edward C. Boynton.

Mr. Boynton read the paper (see p. 1, January issue),

[Communicated After Adjournment by Carl Kinsley.]

In common with all those who have occasion to compare capacities, I am exceedingly interested in the method proposed by Dr. Pupin. Since the time of Maxwell we have understood that absorption is due to the heterogenous nature of the dielectric, but the measurement of the capacity where we have such a dielectric, as for instance, long cables, is difficult and the results still uncertain. The apparatus exhibited was connected as used by the Gott method of measuring capacity when a direct current is employed. This has been found to be probably the most satisfactory of the many possible methods when there is trouble with absorption and leakage. But instead of using a sine wave such as Dr. Rowland (*Philosophical Magazine*, January, 1898) used with his dynamometer zero power methods, an induction coil is employed which gives a wave rich in overtones. The effect of absorption can be expressed by assuming a resistance in series with the condenser. How largely this depends on the period was shown by Dr. Penniman, (*Philosophical Magazine*, January, 1898, p. 70) who found in one case at a frequency of 14.0 complete periods per second an apparent resistance of 139.6 ohms, while at a frequency of 131.1 the apparent resistance was only 5.2 ohms. The importance therefore of knowing the exact periodicity of the current used and of having a high period is apparent.

When the Gott method of measuring capacity is used, the effect of both absorption and leakage is to increase the apparent capacity. The standard, a mica condenser, has negligibly small absorption. The longer the interval of charge is taken the greater becomes this apparent increase of capacity. This is strikingly shown in the case of the following cable, which is entirely typical. Two rubber insulated wires of the cable were used. Their insulation resistance was high. About one-half mile was unarmored and trenched, while one-half mile, which was armored, was lying in New York harbor. The lowest capacity was obtained by a buzzer and telephone. In the second observation the interval was estimated at one-fourth second, the charging key being merely tapped to close the circuit, while the galvanometer remained connected.

From the shape of the curve it is seen that when the time interval is small it must be determined accurately. With commercial condensers of the beeswax-rosin type, the variation of apparent capacity with time of charge is not so pronounced. Certain ones that are rated at one-third microfarad each show that value after a five seconds charge while after a one-fourth second charge their value is 0.292 M.F. and with a buzzer 0.287 M.F.

For scientific accuracy the electromotive force should be a sine function of known period.

When the capacity of cables and condensers for telegraph service is to be determined, the measurement with an alternating

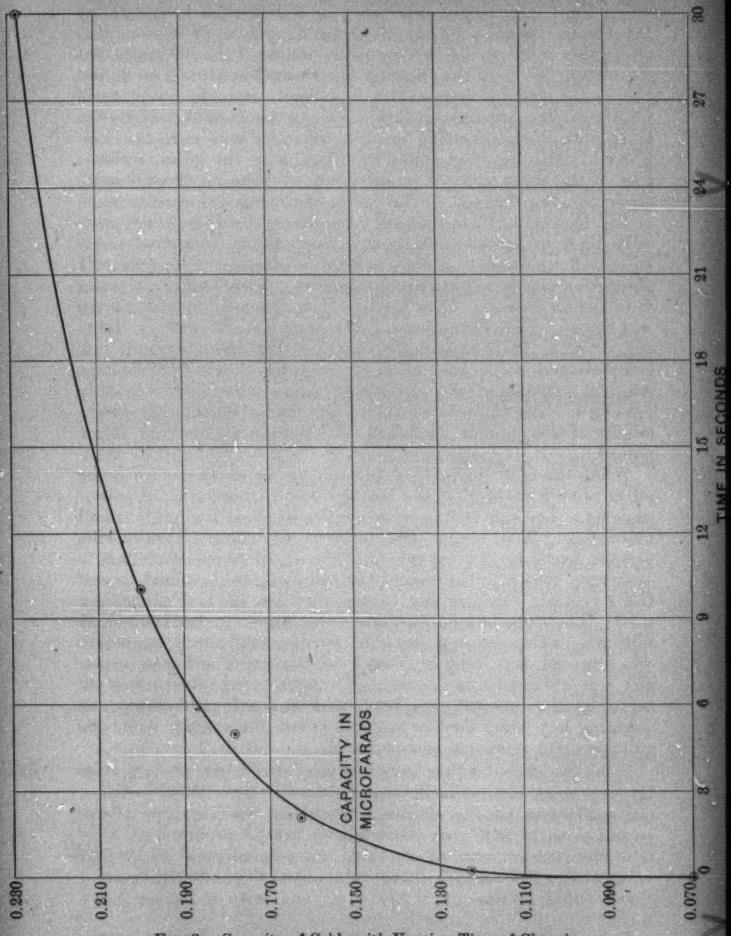


FIG. 8.—Capacity of Cable with Varying Time of Charging.

current will not give the value that must be used in calculations. Uni-directional charging for a suitable time interval will give

better values. Possibly there exists a definite relation between the values obtained by alternating, and those usually found with direct current charging. A careful correlation of the various empirical methods heretofore used and the proposed alternating current methods of measuring capacities would be of great value to both the practical engineer and the laboratory experimenter.

The actual capacity changes but little, if at all, with a change of frequency. In commercial condensers (on alternating current circuits) the heating is usually caused by the absorption. Rosa and Swith (*Physical Review*, January, 1899) came to that conclusion though they quoted Boucherot (*L'Eclairage Electrique*, February 12, 1898) who found that the leakage current alone was responsible. In faulty cables it is quite probable that a good deal of energy will be dissipated by the leakage current. This would have the effect of resistance in parallel with the capacity and it will be unaffected by the periodicity of the current. It may be possible to separate the apparent resistance of absorption from the dielectric resistance by varying the frequency. The alternating current method is the only one that can be used when there is bad leakage. With a buzzer, for instance, when even 5000 ohms of non-inductive resistance is bridged across the cable terminals, already mentioned, the capacity rose to 0.075 M.F. instead of the 0.070 found with the high insulation resistance. No direct current method could be used at all under those trying conditions. The great value of the proposed method of determining the capacity is shown by the preceding illustration and it should be made one of the regular methods for making such measurements.

Governor's Island,
N. Y. Harbor, March, 1900.

A paper read at the 17th Annual Meeting of the American Institute of Electrical Engineers New York, May 15th, 1900. Manager Sheldon in the Chair.

EXPERTS AND EXPERT EVIDENCE.

BY J. E. HINDON HYDE.
Of the New York Bar.

The following notes are simply intended to furnish a subject for discussion, for many of the members of the INSTITUTE are much better qualified from experience to speak on the matter than the author, while the others have probably given the subject more thought from personal interest in it than I, as a lawyer only, have bestowed upon it. My duty to my client and the court has been discharged when I have engaged a competent expert to explain the scientific facts and draw therefrom the proper deductions which aid my client's case, and with his expert aid dispel the illusions called up by my adversary, for my adversary's deductions are always illusions.

But it may be useful to explain what I mean by a "competent expert," for it means much more than that he shall have knowledge of the things about which he is to testify.

The subject is of very great importance to us all—to us lawyers, because it is necessary for us to have experts; to the experts, because it is necessary for them to be competent; to us jointly, because we must devise some way to remove the objections which the courts and the public now undoubtedly have to expert evidence.

The newspapers, in connection with a recent famous trial, have shown to us all that the public, as such, derides expert evidence, while in the decisions of the judges we constantly meet with strong criticisms, not only of the mass of irrelevant matter introduced into cases by the united action of lawyers and experts, but even amounting to doubts of the honesty of the experts.

Such expressions as "the usual conflict of expert evidence" and "you can hire an expert to testify to anything," are not uncommon; while in a recent decision Judge McPherson used the following language:

"There is the usual conflict of expert opinion in the testimony, resulting in the usual doubt in the mind of the Court whether such a thing as scientific truth can be said to exist when a question of infringement is being fought out."

Now, while unfortunately there is much truth in these criticisms, there is undoubtedly something to be said on the other side, and at least we can resort to that expedient of a lawyer with a bad case—abusing the adversary—by calling attention to the fact that judges sometimes err, are frequently overruled, and that even the United States Supreme Court has been known to reverse itself.

But the better way is to try to remove the prejudice against the experts. It is not impossible, for I have in mind now a very good expert who died some years ago and whose word carried weight and respect with the judges, because he was known to be honest and straightforward, and his manner was so persuasive and simple that he generally carried conviction to the men to whom he spoke.

Of all the scientific subjects which enter into the cases to be determined by the courts, I know of none more important than that of which electricians are making a study. Indeed, I think that I may fairly say that at the present time there is none so important unless it be chemistry, and there is a bond of union between them. And yet I think that the members of the Institute will agree with me that there is no branch of science less understood in its essential nature and in many of the various possible means of its application to the service of mankind than electricity, not even excepting psychic force, which, for all that we know, may be only a branch of electro-biology. The limits of electro-genesis are not now definable, while the protoplasm itself, and even the atom, must be theoretically considered as both positively and negatively electrified.

But, without passing into this speculative inquiry, it will be sufficient for my purpose to state that in the essentially practical subjects of electro-dynamics and electro-metrics, with their various industrial branches of electro-chemistry, electro-metallurgy, electric lighting, electric traction, electric heating, electro-

therapeutics and electrography—all covered by the expression, "the translation of electric energy into heat, light and power," there is a vast field in which electricians themselves still have a great deal to learn, while it is one in which the courts have only their guidance in passing upon very important questions of property rights.

When we reflect that it is almost a century since Oersted's fundamental experiment laid the foundation for the practical utilization of the electric current for power purposes, it seems strange that we do not know more about electricity than we do, and yet what little has been ascertained is practically a sealed book to most of mankind, including the judges of our courts.

It is not so very long ago that I had to describe to a prominent judge who was hearing an argument in a most important electrical case what a condenser was, and, further, had to explain to him that the current from a battery could be increased by adding another cell. In dictating an opinion on certain patents and mentioning hysteresis, I almost produced hysteria in the stenographer. And I should very much dislike to-day to have to depend upon "judicial notice" being taken of Ohm's law. What does the great body of lawyers know of "harmonics?" What of Foucault currents? What of the relative merits of single and multiphase motors? While few of them remember Lenz's laws of induction longer than from Saturday night to Monday morning.

Neither is it essential, nor even altogether desirable, that lawyers should know all these matters, for if we did we should not have time to learn law.

As late as May 5th, 1898, one of our judges used the following language:

"The mystery and uncertainty which surrounds everything relating to electricity, and the feeling of admiration, almost akin to reverence, for those men who have subdued this unknown and dangerous force and made it do the world's work, make the Court diffident about applying these principles which are axiomatic in the patent law. There is always the apprehension that injustice may be done through failure to comprehend the abstruse and difficult problems presented."

(*Edison Elec. Light Co.* v. *E. G. Bernard Co.*, 88 F. R., 267.)

It must be apparent, therefore, how completely the judges are at the experts' mercy, and how important it is that they should

be able to rely upon competent experts. When we remember the enormous number of patents which have been granted for electrical inventions and the vast sums of money that have been spent in litigation, the importance is only emphasized; and when it is recalled that the two most important electrical cases that have been decided by the United States Supreme Court, that relating to the telegraph and that relating to the telephone, were both decided by a divided Court, the difficulties of the judges must be evident. This is also shown in another important case. No one who reads the decision of the Circuit Court in the case of the Edison Electric Light Co. v. Westinghouse, Church, Kerr and Co., involving the patent for the Edison Feeder System, (55 F. R. 290), and compares it with the decision of the same case on Appeal (63 Fed. R., 588), can fail to see how differently different judges regard expert testimony. In the one decision, the expert evidence given by the well-known and even celebrated experts for the complainants is treated as of authoritative weight, while in the other decision it is all brushed aside and the Court decided the case on its own assumption of the facts, owing to the conflict of evidence, and held that Edison had not made an invention.

Once more, the judges' difficulties are illustrated in the recently decided case of *Electric Co.* v. *Scott*, 97 Fed. R. 588, on the Tesla patents, where Judge McPherson's opinion consists almost entirely of quotations from the complainants' brief, which, in turn, are extracts from the experts' evidence.

What qualities, then, make a competent expert, and especially a competent electrical expert?

Perhaps it will not be amiss to point out first some tendencies that are to be avoided.

Bacon, in his "Novum Organum," and Herbert Spencer, in his "Study of Sociology," have indicated many of the biases with which men approach the solution of a question. Some of the special biases of an expert may be enumerated.

1. *The bias of his school.*—The importance of this can hardly be overestimated. It is exemplified in the controversy between the allopathic and homœopathic schools in medicine; that between the advocates of the alternating current and those of the direct current, respectively, for certain electrical purposes; the warfare between the Plutonists and Neptunists, in geology; the dispute as to the first discoverer of induction; the fight over the

limits of deduction and empiricism, the old question as to the nature of sight; the discussion between the followers of Galvani and Volta respectively; the former controversy as to whether oxygen or hydrogen should be used as the standard of atomic weight; the different claimants for the suggestion of the theory of magnetomotive force in a magnetic circuit, with its analogies to Ohm's law and its consequent bearing upon the forms of dynamos, and there are many other cases that will occur to every one.

2. *The bias of race.*—I do not believe that even a scientific man is entirely devoid of this prejudice.

Englishmen know that Scheele discovered oxygen, but they always give the credit to Priestley, while Swedes know that Priestley discovered it, but they give the credit to Scheele. No Frenchman speaks of Adams as the discoverer of the planet Neptune, while Englishmen rarely assign the discovery to Leverrier; and Galle of Berlin is the German discoverer. So, too, the telegraph has been claimed for Steinheil, for Wheatstone, for Davy, for Henry, and for Morse.

3. *The bias of personal dislike.*—This dislike may be either to the client or to the lawyer or to the expert on the opposite side, but it should not be allowed to influence the expert.

4. *The bias of ratio of fees to services*—Of this bias I shall not speak, as we all understand it. The poet Gay has spoken of it in speaking of lawyers. He writes:

> "I know you lawyers can, with ease,
> Twist words and meanings as you please.
> That language by your skill made pliant,
> Will bend, to favor every client;
> That 'tis the fee directs the sense,
> To make out either side's pretense;
> When you peruse the clearest case,
> You see it with a double face;
> For skepticism's your profession;
> You hold there's doubt in all expression.
> Hence is the bar with fees supplied;
> Hence eloquence takes either side."

5. *Then there are also the biases due to the "personal equation."*—There are experts whom I have known who seemed constitutionally unable to answer questions directly and simply. Sometimes the bias takes the form of a seeming choice of unnecessarily involved and stilted language, as if the witness were endeavoring to impress the hearer with a sense of the vast learn-

ing of the expert. Again, it is shown in an apparent unwillingness to admit the truth, and if the admission is finally made, it is prefaced by really unnecessary explanations which do not, of course, destroy the value of the admission when finally obtained, but they affect the value of the expert, both to the cause and to the Court.

We can all imagine how provoking such answers are, particularly upon cross-examination. Nor do they do any good—they seem to be a sign of weakness, though they may not be such. Besides spinning out the record to an unreasonable length, and thus tending to irritate the judge who has to read the testimony, or wearying the jurors who have to listen, they add enormously to the expense, both in the way of fees for recording them and in the printer's bill, while the expert who gives such answers incurs the suspicion of trying to increase his own fees, as well as of being unfair. Of course I know that we lawyers, in our ignorance, often put questions that cannot be answered directly, and the expert in such cases is obliged to protect himself by necessary explanations; but it is better not to carry the matter too far, and often on cross-examination it is the best policy to answer "yes" or "no" whenever possible, as such answers do not provide the lawyer with material for his next question. Indeed, they are often the most embarrassing answers that can be given. Again, instead of giving explanations, it is frequently wise to answer "yes and no," where neither assent nor dissent alone would be sufficiently full, for then you put the responsibility of further inquiry into what is often a totally irrelevant subject upon the lawyer and he must take the blame. Besides, it is not necessary always to explain so much, for the judges are not always ignorant, and it must be your experience, as it is mine, that, as a general rule, justice is done in a case by the time, if not before, it has been decided by the highest court. We may not always approve the final decision, but in the great majority of cases it is apt to be right.

Then there are experts who have a special hobby, and who endeavor to bring everything around to that particular pet; and though they may not be quite so bad as a lawyer with whom I am acquainted, and of whom it was said that on one occasion, in addressing the Court, he remarked: "I do not know much of the subject of this case, but I do know something of electricity," (I wonder how much he knew), "and, if the Court please, I will

say something of that," yet they are apt to bring in irrelevant matter in exploiting their own "fads," and, possibly, their own reputations. Then, again, I remember an instance where an expert took seventeen days to answer one question. It is true that it was not entirely his fault, for he was asked by the lawyer who retained him to compare the invention of the patent in suit with each and every of the patents and publications set up in defense, and this, of course, required time. The expert was left alone with the typewriter, without questions to guide him as to what matters were important and what references might be quickly passed over. The proper way was for the lawyer and the expert to have consulted previously and decide which references were the most important, so that the others could be dismissed in a few words. The lawyer should have taken the burden of telling the Court why the references were not all material, and the Court would have understood.

There is another very important defect in the mental make-up of some experts, and that is a tendency to discuss and explain their interpretation of the law. I recall a very able expert whose inclination in this regard was so strong as to frequently interfere with his usefulness. He would not only dispute the matter with the lawyer who called him, but even when told that the United States Supreme Court had definitely decided the question contrary to his contention, he would reply that in that case the Supreme Court was in error. The dispute could generally be kept off the record while he was under direct examination, by a prior understanding with his retaining counsel, but he usually found some place for his views during the cross-examination. All this was wrong. The Judge paid no attention to him on this subject, and he only went out of his province to engender probable irritation and to cause undoubted unnecessary expense.

Now, having stated some of the qualities which an expert should not possess, let me pass to the other side of the picture and enumerate some of the privileges, rights, duties and characteristics which pertain to a competent expert. First as to his privileges.

I can conceive of no case where it is not the privilege of a qualified expert to testify, unless it may be because of his relations to the opposite side, and I shall speak of this subject later.

There is no inherent reason why an expert is disqualified from stating the facts within his knowledge because he thinks that his client has a bad or unjust case.

I cannot do better than to quote some remarks made by Dr. Johnson which Boswell has recorded as follows:

"We talked of the practice of the law. Sir William Forbes said he thought an honest lawyer should never undertake a cause which he was satisfied was not a just one. Sir, said Johnson, a lawyer has no business with the justice or the injustice of the cause which he undertakes, unless his client asks his opinion, and then he is bound to give it honestly. The justice or the injustice of the cause is to be decided by the Judge. Consider, sir, what is the purpose of courts of justice! It is that every man may have his cause fairly tried by men appointed to try causes. A lawyer is not to tell what he knows to be a lie; he is not to usurp the province of the jury and of the judge, and determine what shall be the effect of evidence, what shall be the result of legal argument. As it rarely happens that a man is fit to plead his own cause, lawyers are a class of the community who, by study and experience, have acquired the art and power of arranging evidence and of applying to the points at issue what the law has settled. A lawyer is to do for his client all that his client might fairly do for himself if he could. If by a large superiority of attention, of knowledge, of skill and a better method of communication, he has the advantage of his adversary, it is an advantage to which he is entitled. There must always be some advantage on one side or the other, and it is better that the advantage should be had by talents than by chance. If lawyers were to undertake no causes until they were sure they were just, a man might be precluded altogether from a trial of his claim, though were it judiciously examined, it might be found a very just claim."

I think that the foregoing statement is peculiarly applicable to an expert even if its truth can be questioned in regard to a lawyer; as Lord Eldon did question it on one occasion. For the expert is called upon simply to state facts, and his conclusions from those facts. He is not required to enlist mistaken sympathies in a bad cause by persuasive eloquence. He should not be a "sworn advocate," as he has on frequent occasions been justly called. He may present the facts in the manner most favorable for the side for which he testifies, but they must be

facts and they must be stated so as not to mislead. Each expert must judge for himself in these cases.

Imagine a case of poisoning. Would an expert be justified in stating that he had found arsenic in the subject of an autopsy, and suppress the facts, either that it was in such minute quantities that it could not have been the cause of death, or in such portions of the cadaver as showed that it was injected in the embalming fluid? Would it be right for him to trust to the opposing lawyer to bring out such facts? Again, in the case of an engine, would it be right for an expert to say that it worked, but omit to state the fact that it would only run during an incomplete revolution, or that it ran so badly as to show that it was a failure? Still again, in an electrical case, would an expert be right in asserting his untried theory as a fact? I leave the answers of these questions to the experts.

I was asked this question by an expert a short time ago: "If I am retained in a case by a lawyer, and, after looking into it, find that I cannot take the position that he wishes me to take, am I entitled to keep my retainer as compensation for my labor of examination, and refuse to testify? Further, can I accept a retainer from the other side and testify for them?

This looks like a complicated question, but it seems to me that the answer is really simple.

I reply that an expert is certainly entitled to compensation for his labor in examining the questions in a case in which he is asked to testify, if he has to expend labor in such examination. Whether he shall keep the whole amount of the retainer which was given to him in the expectation that he would testify, depends upon the amount of the retainer, and the amount of labor necessarily expended in the preliminary examination of the subject. Although it is not always just to the expert, I think that the better policy would be to return a considerable part of the retainer and keep the rest as compensation. I know of one expert who used to return the whole of the retainer if he was unable to act, but he was a rich man and could afford to be generous. Even then, I believe that his practice tended to his advantage in the end.

The better way is to have an understanding in advance that a fixed sum is to be paid for the preliminary examination, making this sum large enough to compensate also for the expert's risk that such examination may disclose facts to him that would pre-

vent him from testifying for the other side, even if he found that he could not act for the first applicant for his services, and, perhaps, a further sum as retainer, if he testifies.

The answer to the second part of the question is as follows:

If the expert has, during his preliminary examination, learned facts which can be even remotely said to be confidential and which are detrimental to the side for which he was asked to testify, he cannot in any case act for the opposite side, for he would be under a moral obligation to tell all that he knows that would assist the whole truth being elicited. If he has learned nothing which is of a confidential nature, I know of no reason why he should not testify for the other side.

The following question has also frequently arisen. An expert has testified in a case, or several cases, where it was necessary to learn much of the client's business, his machinery and processes. In a new suit between the client and a new antagonist or one of the old antagonists, he has not been retained by his old client for some reason, but the adversary seeks to retain him. Is he at liberty to accept?

I think the proper thing for the expert to do in such a case is to state the facts to his old client and offer him his services, and if they are declined, ask permission to act for the other side. This is not indelicate, and it is only fair. The client can be trusted to either retain him or give him that permission. If, however, he does neither, the expert must judge for himself. It is certainly not fair that he should be forever tied up because he has once acted for a client.

A celebrated lawyer once laid down this rule of action for himself.

"If I ever had any connection with a cause, I will never permit myself (when that connection is for any reason severed) to be engaged on the side of my former antagonist. Nor shall any change in the former aspect of the case induce me to regard it as a ground of exception. It is a poor apology for being on the other side, that the present is but the ghost of the former cause.

I think that that lawyer went a little, but very little, too far.

Now, a few words as to the duties of an expert. Some of these have necessarily been referred to above, but there are one or two others that may be mentioned. First and foremost of these I put honesty, unswerving honesty and truthfulness.

Honesty and truth to the client, honesty and truth to the lawyers, and honesty and truth to the Court. In no other way can a man secure the respect of others, and, more important still, his self-respect.

In this way he will not convey false or misleading impressions, he will not suppress a part of the truth, and he will not "pad out" a record for the sake of increasing his fees, will not waste time, nor will he lend his aid to any of these things. He will govern himself with equal strictness on direct examination and on cross-examination, and he will take a position the reverse of that of the man who said: "If the facts do not agree with me, so much the worse for the facts."

Again, these qualities will lead him to answer each question by itself as it is presented, and not concern himself with what he may have previously said in that or in other cases, or on other occasions. It is probable that he has made former mistakes, but they must be confessed without fear. No explanation by him will ever reconcile two *inconsistent* statement of *fact* so as to cause belief that both statements were correct. He may show that they are not inconsistent, if he can, but this should be done as briefly as possible. If they are inconsistent, it is his duty to confess it.

Another duty is to be fully qualified to speak authoritatively on his subject. It would be well for him to remember what a Judge once said to me in explaining why courts are so jealous in assuming jurisdiction of a case. "It is not," he said, "that the judges are attempting to shirk work, but every case involves personal or property rights, and unless the law clearly declares that it is his duty to do so, a conscientious judge always dislikes to assume the responsibility of curtailing the one or taking away the other."

It is the expert's duty to his client and his lawyer to show the same hesitation unless he is fully equipped for the case.

My opponent once lost a very important electrical case against the city of New York, so that he had to drop it before it went to Court, because his expert had not informed himself about what he had to testify before he went upon the stand. The expert explained to me afterwards that his client would not meet the expense, but he himself evidently did not see that he should have declined to act without this necessary knowledge, and that even if it could not be said that he was attempting to deceive the

Court as to his qualifications, he was certainly jeopardizing his client's interests. This is a clear illustration of the necessity of a fully qualified expert.

Again we can all appreciate the value of an answer which reads, "I know it because I have tried it," and how such an answer, even when given by a young expert, counterbalances all speculative answers made by much more celebrated men.

I recall a curious instance which shows the necessity for experiment. A young electrical expert was asked on cross-examination for an explanation of the action of a certain structure and he gave his answer, basing it on his general knowledge of what had been demonstrated by Faraday. but without having had an opportunity to prove his correctness by experiment. He was contradicted and even derided by the opposing and better-known expert, and yet subsequent experiment showed that he was right. But it was too late for that case, and the Judge probably gave more weight to the erroneous view of the better known man than to the other.

As has been said by a well-known writer: "In deciding between contradictory expert testimony, juries should consider the respective reasons, ability, knowledge and fairness of the experts. To judge according to their number or their fame would be unsafe. The wealthier litigants are generally those who employ the more numerous and the more expensive expert witnesses. But it is not always the wealthier litigant who is right in a controversy, nor always the more famous expert who is right in his opinion," (he might have added his facts). "The carefully digested views of a young and studious scientist may be often more nearly true than the more hastily formed opinion of a more experienced man." (Walker on Pats. 3rd Edit. 498.)

There are many matters in which the Court or jury is not competent to weigh the "reasons," "ability," and "knowledge" of experts, but it can always weigh their "fairness"; and the man who hesitates to speak before he has proof of what he says always outweighs the man who boldly asserts a thing without having such proof.

Once more, an expert's duty to the Court should govern his action toward the lawyer who retains him. As an expert's first duty is to the Court as the representative of justice, he must not allow any consideration to interefere with this duty. With this in mind, he will primarily wish the ques-

tions upon which he is to testify to be submitted to him in such a manner that he can formulate his own conclusions without unavoidable bias, and without persuasion. It is best that he should not know, if possible, whether he is to act for the complainant or for the defendant, and he will ask the lawyer not to suggest anything that will tend to influence his opinion before he has made his preliminary examination of the matter in dispute. It is, perhaps, too much to expect that the lawyer will not generally allow his zeal to suggest what he wishes, but I have known cases where this was not done, and we can all readily imagine the feeling of relief with which the experts then went on the stand and testified that their answers had not been influenced by any considerations other than their own independent thoughts. I am sorry to have to say that these occasions are rare, but there can be no doubt that they should be universal. If they were known to be such, I think that in a large preponderance of cases, there would be no "conflict of expert evidence."

The experts have this matter in their own hands, and it is not only their duty, but their right to see that the practice is general.

And now in conclusion, a few words as to the characteristics of a competent expert.

I am pleased to be able to state that in my experience I have rarely met with an expert who was other than courteous, modest, and tolerant in manner. They seem to have an innate perception of two valuable rules laid down by Vice-Chancellor Hoffman for the guidance of lawyers. The first of these is as follows:

"Should I attain to that eminent standing at the Bar which gives authority to my opinions, I shall endeavor in my intercourse with my junior brethren, to avoid the least display of it to their prejudice. I will strive never to forget the days of my youth, when I, too, was feeble in the law, and without standing. I will remember my then ambitious aspirations (though timid and modest), nearly blighted by the inconsiderate or rude and arrogant deportment of some of my seniors; and I will further remember that the vital spark of my early ambitions might have been wholly extinguished, and my hopes forever ruined, had not my own resolutions and a few generous acts of some others of my seniors raised me from my depression. To my juniors, therefore, I shall be ever kind and encouraging, and never too proud to recognize that, on many occasions it is quite probable that their knowledge may be more accurate than my own, and

that they, with their limited reading and experience, have seen the matter more soundly than I, with my much reading and long experience."

We all readily see how this is as applicable to experts as it is to lawyers.

The second rule reads:

"Should judges, while on the bench, forget that as an officer of the court, I have rights and treat me even with disrespect, I shall value myself too highly to deal with them in like manner. A firm and temperate remonstrance is all that I will ever allow myself."

This may be paraphrased for experts as follows:

Should lawyers, while questioning me, forget that as a witness and a gentleman I have rights, and treat me even with disrespect, I shall value myself too highly to deal with them in like manner. A firm and temperate remonstrance is all that I will ever allow myself.

There is no doubt that occasions frequently arise when there is a strong temptation to turn foolish questions into ridicule, when a trifling advantage may seem to be gained by sarcasm or an unkind witticism, but this does not pay in the end. Front and foremost of all, the sacredness of the cause should be borne in mind; it is a serious matter that calls for expert evidence, and the expert should be serious. Jokes are very good fun, but unless they illustrate a point you wish to make, they are out of place in a court of justice.

I look forward to the time when the Court will have its own expert as its technical adviser, and then, while there may still be that "conflict of evidence" which is now so usual, it will do less harm. The Court and its expert will work together to arrive at the truth and sift the chaff from the wheat. I believe that this practice has been adopted in France and in Germany.

Finally, when I add that the competent expert tries to explain the most technical questions in the least technical way; that he is equipped with a wealth of illustrations to illuminate the facts; that he confines his answers as closely as possible to the questions; that he is not timid in admitting his ignorance; and, besides all this, is ingenious in devising models and experiments to explain his statements, I believe that I have enumerated most of the qualities which a competent expert should possess.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Report of Council for the Fiscal Year Ending April 30, 1900.

As required by the Constitution, the Council submits herewith for the information of the membership a report of its work; also the financial standing of the Institute at the close of the fiscal year, April 30, 1900.

Four meetings of Council and six of the Executive Committee, constitutionally representing the Council, have been held during the year.

The completed work of the Committee on Standardization, published in pamphlet form for general circulation, has received extended endorsement, and the number sold indicates that it is being used in practical work. This committee is now co-operating with a committee appointed by the American Society of Mechanical Engineers for bringing about a standardization of capacities, sizes, speeds, etc., between direct connected steam engines and electric generators.

For several years past the Council has reported certain funds as being deposited with different trust companies. During the past year it being evident that such corporations were not in all cases beyond suspicion, it was deemed advisable to withdraw from deposit all surplus funds and invest the entire amount in Government bonds. These 3 per cent. bonds were purchased, $4,000 at 112 and $1,000 at 111½, commission $2.50. During the year $2,000 had been diverted from the current bank account, in order that it might be placed at interest. This amount constitutes simply a reserve fund, and was established because the bank account was unnecessarily large, and also that the interest might cover the exchange now charged on country checks, which it has done, the accrued interest during a portion of the year amounting to $17.18, while the bank exchange was $6.90.

In accordance with a vote of the Executive Committee May 16, 1899, the Commercial Fund existing was closed, and the cash balance of $289.29 was transferred to the treasurer's general fund. The items formerly appearing in that account are now included in the regular financial report.

By direction of the Executive Committee, the Secretary and Chairman of the Committee on Papers and Meetings compiled and issued to the membership a Handbook, which has proved to be a useful publication, especially for the information of candidates for membership.

As the result of correspondence with the Institution of Electrical Engineers a committee of three was appointed by direction of Council November 22, 1899, to ascertain the probable number of members who would visit Europe this year, and who would probably attend a joint meeting of the two societies should it be arranged for. Favorable replies being received from seventy members, Mr. T. C. Martin, who sailed for London and Paris in March, was authorized to represent the Institute in arranging for a joint meeting at Paris on August 16th.

In order that the files of periodicals might be made more accessible for reference the Council has appropriated five hundred dollars for the erection of two library stacks in the offices of the Institute and the binding of about three hundred volumes.

At the meeting of Council March 28th a committee was appointed to

represent the Institute in encouraging the establishment by the Government of a Standardizing Laboratory.

The Board of Examiners has held nine meetings during the year, at which forty-five applications for transfer were considered, twenty-six being recommended for transfer, eleven held over for further information and eight disapproved.

The total membership at the close of last year's report was 1,133, classified as follows:

Honorary Members	2
Members	363
Associate Members	768
Total	1133
Associate Members elected May 1st, 1899, to April 13th, 1900	130
" " " previous year and since qualified	10
Total	1273

Resignations have been received during the year and accepted from the following members who were in good standing:

MEMBERS:

PARK BENJAMIN, HAROLD BINNEY,
ROMAINE CALLENDER, GEORGES D' INFREVILLE,

ASSOCIATE MEMBERS:

JAMES S. ALDEN, FRED'K. MACKINTOSH,
HENRY W. CARTER, ERNEST MERRITT,
JULIAN DUBOIS, EVERETT MORSS,
FRANCIS M. DYER, R. H. MACMULLAN,
FRANK W. GLADING, ALBERT SCHEIBLE,
R. N. LARRABEE, DANA C. WELLS.

Total Resignations 16

There have been the following deaths during the year:

MEMBERS:

S. DANA GREENE, JAMES HAMBLET.

ASSOCIATE MEMBERS:

WILLIAM F. KELLY, CHAS. T. KITTENHOUSE,
ALEXANDER STRATTON.

Total deaths	5
Dropped as delinquent	50
Elected but not yet qualified	19
	90

Leaving a total membership of 1183 on April 30th (a net gain of 45), classified as follows:

Honorary Members	2
Members	374
Associate Members	807
	1183

A list of associate members elected during the year accompanies this report, and will also appear in the TRANSACTIONS.

The reports of the Secretary and of the Treasurer show in detail the financial standing of the INSTITUTE at the close of the fiscal year, together with an itemized statement of receipts and disbursements during the entire year:

SECRETARY'S BALANCE SHEET

FOR THE FISCAL YEAR ENDING APRIL 30, 1900.

Dr.		*Cr.*	
Balance from previous year	$ 634 36	By cash to Treasurer	$13,089 53
Receipts for the year	12,711 65	Balance to next year	256 48
	$13,346 01		$13,346 01

ITEMIZED STATEMENT OF RECEIPTS AND DISBURSEMENTS OF THE INSTITUTE

FOR FISCAL YEAR ENDING APRIL 30, 1900.

GENERAL ACCOUNT.

Receipts.		*Disbursements.*	
Treasurer's balance from previous year	$2,664 46	Revenue Tax	$12 79
Secretary's " " " "	634 36	Bank Exchange	6 00
Entrance Fees	650 00	Chicago Meetings	51 00
Life Membership Fees (G. H. Stockbridge, James Mitchell, C. O. Mailloux, F. W. Hadley)	400 00	Ice	8 09
Past Dues	979 75	Laundry	24 35
Current Dues	9,031 37	Office Expenses	7 50
Advance Dues	93 50	" Fixtures	34 15
Standardizing Reports	35 80	Duties	40 21
Electrotypes	30 14	Express	60
Transactions Sold	508 66	Telegrams	181 58
" Subscriptions	284 59	Stenography and Typewriting	22 95
Advertising	60 00	Stationery and Miscellaneous Printing	830 70
Received for Binding	52 25	Postage	699 75
" " Congress Book	4 20	Messenger Service	460 19
" " Reprint Vol. IV	2 00	Salary Account	9 45
Badges	250 90	Meeting Expenses	2,580 00
Certificates	30 00	Rent Offices and Hall	261 60
Cash Balance from Commercial Fund	289 99	Engraving	1,100 00
		Binding	428 14
		Publishing Transactions	551 68
		Badges	2,434 27
		Certificates	236 84
		Reserve Fund	12 00
		Compounded Membership Fund	2,000 00
		Appropriation for Secretary, London and Paris	1,132 17
		Secretary's Balance to next year	500 00
		Treasurer's Balance to next year	256 48
			2,205 08
	$16,010 47		$16,010 47

All outstanding bills against the INSTITUTE were paid in full April 25th.

There is due the INSTITUTE and probably collectible, $1422.41

The amount reported last year as probably collectible was $805. The amount actually collected was $979.75.

The total receipts for the past year exceeded the previous year by $1,223.69, while the expenses were $1,353.11, showing an excess in growth of expenses over the gain in receipts of $129.42. The net financial gain during the year was $2,005.62.

Property on hand according to inventory, May 1st, 1900.

Office Furniture and Fittings	$ 276.65
Badges	127.00
Catalogue Type and Cases	203.37
Library	300.00
Transactions on Hand	3,293.50
Congress Books	537.00
	$4,737.52

TOTAL NET ASSETS.

Treasurer's Cash Balance	$2,205.08
U. S. Government Bonds	5,000.00
Secretary's Cash Balance	256.48
Property as per Inventory	4,737.52
	$12,199.08
Total assets last year	10,770.49
Increase	$1,428.59

Respectfully submitted for the Council,

RALPH W. POPE,
Secretary.

TREASURER'S REPORT.

FROM APRIL 30, 1899, TO MAY 1, 1900.

GEORGE A. HAMILTON, TREASURER, in account with AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Dr.

Balance from April 30, 1899	$ 2,664 46	
Received from Secretary, April 30, 1899, to May 1, 1900	13,089 53	$15,753 99

Cr.

Disbursements from May 1, 1899, on warrants from Secretary as approved by Council or Executive Committee Nos. 1105 to 1299 inclusive	$13,548 91	
Balance to new account	2,205 08	$15,753 99

The following invested funds appeared in last years report:

Building Fund, Mercantile Trust Co	$ 990 48	
Compounded Membership Fund State Trust Co	1,413 44	$2,403 92
Accrued interest		41 73
To which was added fees of G. H. Stockbridge and James Mitchell		200 00
Reserve Fund created during the year with accrued interest		2,017 18
Balance appropriated from General Fund for Bond purchase		932 17
		5,595 00

This total of $5,595 has been invested by direction of Council in U. S. three per cent. bonds in the name of the American Institute of Electrical Engineers and deposited in its safe, apportioned as follows:

	FACE VALUE	MARKET VALUE.
Building Fund	$ 1,000 00	$ 1,090 00
Compounded Membership Fund	2,000 00	2,180 00
Reserve Fund	2,000 00	2,180 00
	5,000 00	5,450 00
Treasurer's balance carried forward to next year	2,205 08	
Total cash and bonds	7,205 08	

Respectfully submitted,

GEO. A. HAMILTON,
Treasurer.

New York, May 15th, 1900.

TELLERS' REPORT.

ANNUAL ELECTION MAY 15TH, 1900.

FOR PRESIDENT.

Total Number of Votes Cast.....576.

Carl Hering	381	C. S. Bradley	6
M. I. Pupin	179	L. B. Stillwell	2
C. P. Steinmetz	7	T. D. Lockwood	1

Total.....576

FOR VICE-PRESIDENTS.

Total Number of Votes Cast.....1723.

G. S. Dunn	544	Louis Bell	1
A. V. Abbott	539	Charles Blizard	1
W. L. R. Emmet	526	H. S. Carhart	1
F. A. C. Perrine	12	C. C. Chesney	1
C. O. Mailloux	11	O. T. Crosby	1
C. S. Bradley	9	Charles R. Cross	1
D. C. Jackson	6	Alex Dow	1
Herbert Lloyd	6	I. H. Farnham	1
B. J. Arnold	5	Charles Hewitt	1
J. J. Carty	5	E. E. Higgins	1
W. J. Hammer	5	J. P. Jackson	1
Carl Hering	5	F. W. Jones	1
C. L. Edgar	4	A. E. Kennelly	1
C. P. Steinmetz	4	T. C. Martin	1
M. I. Pupin	3	Wm. Maver, Jr.	1
C. W. Rice	3	W. L. Puffer	1
H. L. Webb	3	Samuel Sheldon	1
F. B. Crocker	2	L. B. Stillwell	1
W. E. Goldsborough	2	C. A. Terry	1
F. A. Pickernell	2	W. D. Weaver	1
A. L. Rohrer	2	S. S. Wheeler	1
H. J. Ryan	2	J. G. White	1
F. Bedell	1		

Total.....1723.

FOR MANAGERS.

Total Number of Votes Cast.....2294.

W. S. Barstow	553	J. I. Aver	1
R. D. Mershon	530	F. B. Badt	1
C. W. Rice	534	J. B. Cahoon	1
C. T. Hutchinson	319	H. B. Coho	1
J. J. Carty	261	C. D. Crandall	1
T. Wolcott	19	Alex Dow	1
R. B. Owens	6	J. J. Flather	1
William Stanley	6	W. E. Goldsborough	1
B. J. Arnold	4	W. S. Hadaway	1
J. B. Blood	4	C. D. Haskins	1
Charles Cuttriss	4	F. V. Henshaw	1
E. J. Berg	3	Carl Hering	1
A. E. Kennelly	3	J. W. Howell	1
T. C. Martin	3	W. J. Jenks	1
F. A. C. Perrine	3	P. M. Lincoln	1
H. F. Albright	2	R. T. Lozier	1
F. W. Darlington	2	S. G. McMeen	1
William J. Hammer	2	F. S. Pearson	1
D. C. Jackson	2	C. W. Pike	1
H. W. Leonard	2	A. E. Merrill	1
C. P. Matthews	2	Thorburn Reid	1
William Maver, Jr.	2	Jos. Wetzler	1
A. L. Riker	2	J. G. White	1
A. H. Armstrong	1	A. J. Wurts	1
W. S. Andrews	1		

Total.....2294.

FOR SECRETARY.

Total Number of Votes Cast.....576.

Ralph W. Pope.....	573	C. T. Hutchinson.....	1
G. S. Dunn	1	Thorburn Reid.....	1
Total.....		576	

FOR TREASURER.

Total Number of Votes Cast.....576.

George A. Hamilton.....576

The total number of voting envelopes handed to the tellers by the Secretary was 666. Of these there were rejected 51, as follows: Unidentified 24, duplicates 25, one vote by proxy (ruled out by Chairman), and one vote of a member whose resignation had already taken effect. In the case of duplicates the one bearing earlier date was rejected according to the established custom of the Institute. This left 615 envelopes to be opened. After opening these, 32 ballots were found to be loose and unenclosed in any inner envelope, and in seven more the inner envelope was signed by the voter. These 39 were also rejected according to the established custom of the Institute, leaving 576 ballots to be counted.

Respectfully submitted,

TOWNSEND WOLCOTT,
SAMUEL REBER,
Tellers.

DIED.

WEISE.—At Princeton, Ill., May 19th, 1900, of heart failure, Will Morrison Templeton Weise, Manager of the firm of Weise Brothers, electrical contractors, Davenport, Ia. Mr. Weise was born in Princeton, January 18th, 1865, where he was educated in the public schools, and subsequently engaged in electrical work in Seattle and Chicago. In 1897 he entered into partnership with his brother at Davenport. He was elected an Associate Member of the INSTITUTE August 13th, 1897.

A paper read at the 17th General Meeting of the American Institute of Electrical Engineers, Philadelphia, May 16, 1900, President Hering in the Chair.

A NEW TRANSMISSION DYNAMOMETER.

BY W. ELWELL GOLDSBOROUGH.

The dynamometer described in this paper is the outcome of a series of attempts to devise an instrument for the measurement of mechanical power comparable with electrical instruments as regards simplicity of design, ease of calibration, permanency of adjustment and the accuracy of the readings obtained. The dynamometer cannot be said to embody any radically new ideas. The principles upon which its success depends have been used in several other instruments; there is presented here simply a new adaptation of these principles. The instrument has been jointly developed by my as-istant, Mr. E. T. Mug and myself.

The dynamometer, as shown in Figs. 1 and 2, is designed especially for testing electric motors of from five to 25 horse power capacity, and can only be used by directly connecting it between the driving element, the motor to be tested and the driven element, a dynamo, adjusted to absorb the power developed. It consists of a shaft, shown in Fig. 3, which is made in two parts. These are kept in line by having a portion of the left hand section turned down to exactly fit a concentric hole bored in the right hand section of the shaft. The joint presents a sufficient bearing surface to enable the instrument to stand quite severe lateral strains, although it is not sufficient to admit of the dynamometer being used to transmit power through the medium of belt-driven and driving pulleys attached to the outer ends of the shaft. For the dynamometer to be available for this class of work, the two sections of the shaft should be made in the form of quills to fit a second shaft extending completely through both sections.

Each section of the shaft is provided with a spiral grip-thread, turned to fit a spiral spring of $\frac{1}{4}''$ pitch. These threads

are shown in the lower half of Fig. 3. The sections are also fitted with jaw collars, shown in the upper half of Fig. 3, that are turned to the same pitch. They have their spiral jaw surfaces cut to overhang the shaft, and make an angle of 45° with the shaft surface. The jaw collars, when forced into position by the lock nuts, with the aid of a spanner wrench, hold the end turns of the springs very securely, preventing them not only from slipping around the grip-thread, but also from springing out when the direction of rotation is such as to tend to unwind the spring.

All the spirals used with the dynamometer must have a pitch of $\frac{3}{4}$ inch, and be wound to snugly fit a shaft $2\frac{1}{32}$ inches in

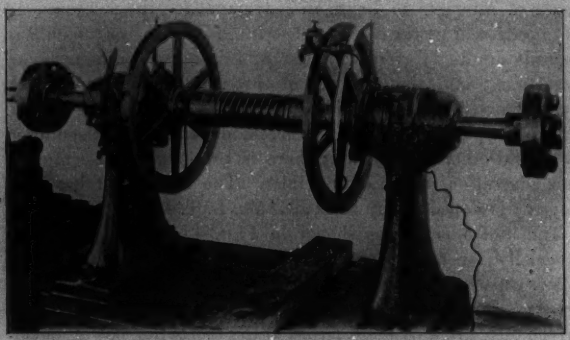


FIG. 1.

diameter. The jaw will securely hold springs from .2 inch to .6 inch in diameter, but necessarily the larger springs give a more stable adjustment and add stiffness to the instrument, considered as a transmitting device. Within the capacities of the springs, therefore, comparing them relatively to one another, the conditions for accurate work are more favorable with the larger springs. All the experimental work referred to here, however, was carried out with a spring made of standard No. 5 steel wire, which is slightly over .2 inch in diameter.

On the free sections of the shaft, shown between the hexagonal sections and the oil grooves, are mounted two instantaneous contact wheels, insulated from the shaft by hard fibre bushings. Each of these wheels, which are shown in Figs. 1 and 2, carries

at its rim a contact edge made of sheet brass and set parallel to the shaft in a hard fibre plug, driven into an inverted V groove, and turned down flush with the surface of the wheel. The contact points are grounded on the shaft, and are therefore in electrical connection with one another, although insulated from the contact wheels. Two brush holders mounted on the dynamometer bearings, and carrying 90° graduated scales, supply the brush supports and complete the contact devices; they provide for the possibility of obtaining readings when the angular deflection of one wheel rim contact point is 180° from the other. The brushes are carried on insulated brush rods and are con-

FIG. 2.

nected with one another through several primary cells and a telephone receiver, all in series. This adjustment supplies the means of detecting when the brushes are so placed that the contact points on the wheels pass under them at the same instant, since at such times a "click" is heard in the telephone at each revolution of the dynamometer shaft.

The description thus far given indicates with sufficient clearness the method of using the dynamometer and of obtaining instantaneous readings of the torque exerted by the driving element at any time. When, in the case illustrated, the motor is started up, the angular deflection between the contact points is

increased above its initial value until the resistance to flexure offered by the spring overcomes the frictional resistance of the generator, and motion is imparted to its armature. If, by speeding up the motor or loading down the generator, conditions are varied so as to increase the resistance to motion between the two machines, there will be a still further deflection of the spring until the balance is restored between the forces acting. The variation in the angular deflection is quickly and easily followed by shifting the contact brushes until the "click" occurs in the telephone. In case the "click" is not sufficiently pronounced, a condenser, shown in Fig. 2, is connected in parallel with the telephone receiver, and its capacity adjusted until the "click" is sharp and clear cut. If ordinary conditions of constant load are maintained, such as are usual in any laboratory test, the "click" in the receiver will be kept up at a constant frequency,

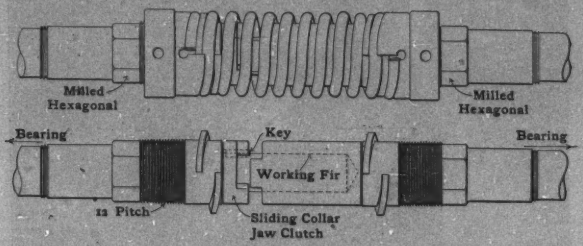


FIG. 3.—Details of Shaft and Spiral Spring Clutch Jaws.

and the relative motion between the contact points will be zero, as shown by repeated trials, even when the smallest spring is used; and when a suitable tachometer is at hand, ideal conditions are presented for making instantaneous determinations of the power transmitted.

In testing a given motor, the best results have been obtained by using a spring which will give a deflection of 180° when the motor is fully loaded. The brush setting admits of an accuracy within .2 of one degree, and thus the determination of the full load torque can be obtained within one-tenth of one per cent. This compares favorably with the results obtained with the usual laboratory electrical instruments.

The method used in calibrating the dynamometer is illustrated in Fig. 2. It is rapidly and easily effected with the aid

of the two long wrenches shown in the cut. These are adjusted on the hexagonal sections of the shaft, one being used as a scale arm and the other as a lever. For any given point the lever is drawn over till the scale arm shows a tendency to raise from its horizontal position; the brushes are then placed on the contact points and adjusted till a clear "click" is obtained in the receiver on making and breaking the battery circuit; and a record is made of the scale arm moment and the deflection scale reading. The calibration curve of the dynamometer fitted with

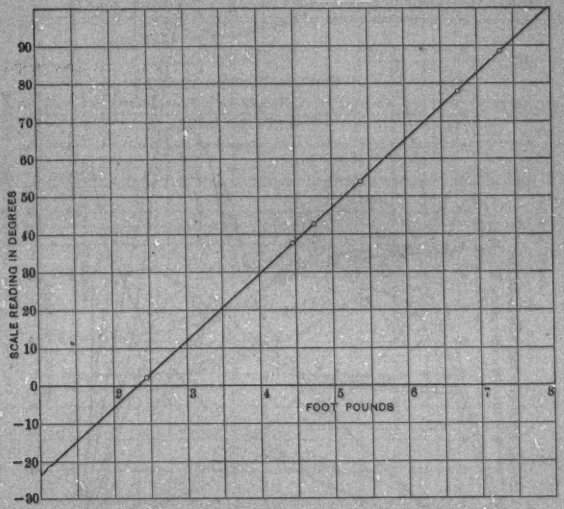


FIG. 4.—Calibration Curve of Dynamometer Spring.

the lightest spring is shown in Fig. 4, and repeated tests before and after use fail to show any change in its calibration constants.

As an evidence of the ease with which accurate results can be obtained, I give here the results of a test made upon a three-phase induction motor conjointly with Mr. Mug, and two seniors in electrical engineering at Purdue University, Messrs. A. F. Chamberlain and C. R. Dooley, to determine the characteristics of the motor. The results may be the more interesting in view of the fact that the test embodied a very simple method

for obtaining the mechanical torque developed by the motor at any speed.

The electrical readings were obtained by connecting an ammeter and the current coil of a wattmeter in each circuit, and a voltmeter and the voltage coil of a wattmeter between each terminal of the motor and the neutral point of the three star connected transformers feeding the motor. The voltage was

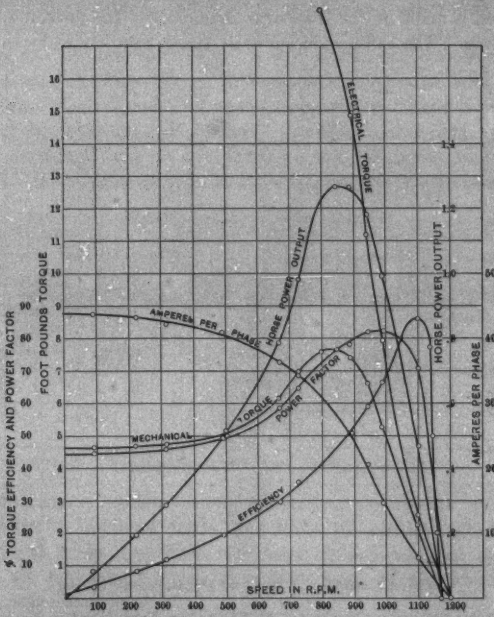


FIG. 5.—Speed Characteristics of Three phase Induction Motor.

kept constant at 53.7 volts between the motor terminals. The speed was determined by belting a calibrated magneto-generator to the motor shaft and reading the voltage developed at its brush terminals; and the torque from the dynamometer. Every factor entering into the test could, therefore, be obtained simultaneously and very accurately.

The motor was started up from rest and held at a low speed by the generator. The generator was connected with its two

shunt field coils in parallel, instead of in series, across its brushes. With this connection it acted as a short-circuited series direct-current dynamo. To add stability to the adjustment, the series coil of the dynamo was separately excited with a small current. The speed of the motor increased till the counter torque developed by the generator equalled the torque of the induction motor. The speed then assumed a perfectly

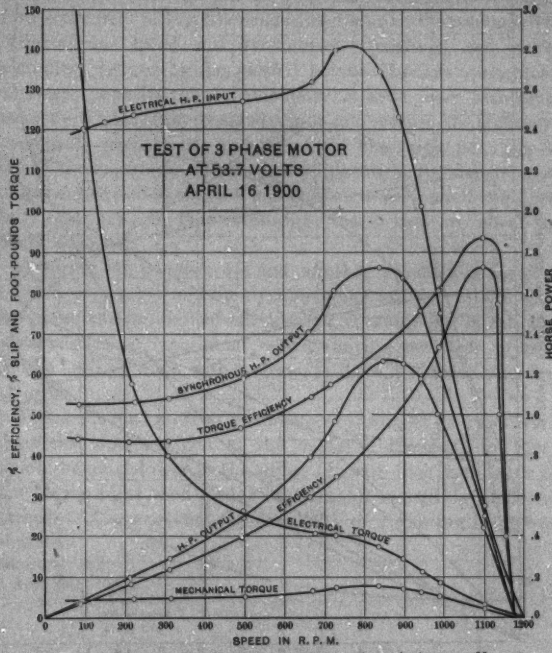


FIG. 6.—Torque Characteristics of Three-phase Induction Motor.

constant value, and readings were obtained from all the instruments. Next, a resistance was connected in the armature circuit in series with the shunt winding; the induction motor increased its speed until the counter torque of the generator again equalled the torque of the motor; the speed again became perfectly constant, and readings were taken. The speed stability of this arrangement is owing to the fact that the torque of the direct

current machine increased very nearly as the square of its armature speed, and therefore very much faster than the torque of the induction motor. Using this adjustment, the speed and torque of an induction motor can be held at all common values at will.

I hope I may be pardoned for dwelling upon these experi-

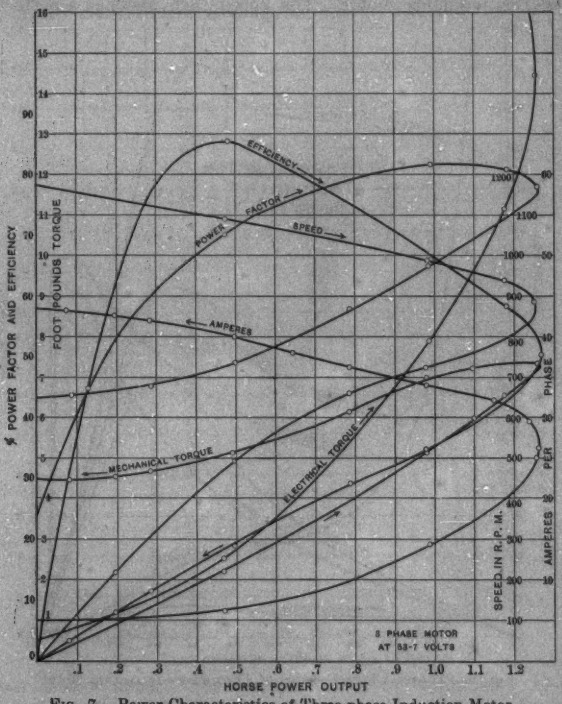


FIG. 7.—Power Characteristics of Three-phase Induction Motor.

mental details, but I do not now recall having ever seen a description in print of a simple and successful method of obtaining the torque of an induction motor for speeds below the speed of maximum torque.

The curves of Figs. 5, 6 and 7 show the experimental values of all the characteristics of the motor below as well as above

the point of maximum power, and bring out the relations that hold between the several factors at the very low speeds. The curve designated "electrical" torque represents the torque that the machine would develop with the given input if it had an efficiency of one hundred per cent. at all loads.

The curves are chiefly valuable in showing the accuracy with which torque readings can be obtained. The curve of "Horse Power Output" of Fig. 5 depends directly upon the product of the instantaneous speed and dynamometer readings, and is very accurately defined by the plotted points. In tests upon motors of larger capacity the dynamometer is even more satisfactory from the experimenter's standpoint; not because its accuracy is necessarily greater, but because of the more rigid coupling between the machines. There is nothing about the dynamometer to get out of order, the bearing friction is very small and practically eliminated in the calibrating; the springs are cheap and easily adjusted.

A spring can be taken out and another substituted for it in about five minutes, and the dynamometer can be calibrated in five minutes; but best of all, perhaps, it can be put in the hands of inexperienced experimenters, after having once been adjusted, and be used by them with success, as there is nothing to get out of order.

When the load on the dynamometer is subject to rapid variations, the clutch, shown at the center of the shaft in Fig. 3, should be moved over to the right. It will then lock with the jaw on the right hand section of the shaft in case the deflection exceeds 90° and prevent the possibility of the spring being overstrained.

Lafayette, Ind.,
April 30th, 1900.

Report of Committee on Units and Standards.

To the Council, American Institute of Electrical Engineers.

Gentlemen:—Your Committee on Units and Standards has considered the following resolution passed at the last regular meeting of the Institute on the 28th of March, 1900:

Resolved: That the Committee on Units and Standards be requested to investigate and report at the ensuing meeting in regard to the advisability of the following:

1. The giving of names to the Absolute Units of the Electrostatic and Electromagnetic systems.
2. The denotation by means of prefixes of multiples of such units
3. The rationalization of the present system, by means of taking the absolute unit of magnetism as equal to the present magnetic line, and the absolute difference of magnetic potential as equal to the present absolute unit of current-turn.
4. The advisability of taking up any or all of the above matters at the Congress to be held in Paris this year.

The Committee desires to report as follows:

1. We consider that there is need for names for the absolute C. G. S. units in the Electrostatic and the Electromagnetic systems; also for suitable prefixes to denote decimal multiples and submultiples of these units, in supplement and addition to those already in common use.
2. That the International Electrical Congress convening this year at Paris should be urged to bestow the above mentioned names and create said decimal prefixes.
3. That much advantage would accrue to a universal "rationalization" of electric and magnetic units, and that the Congress be requested to consider the means and advisability of such "rationalization."
4. That we recommend that the whole subject should be brought up as a topic for general discussion at the approaching General Meeting of the Institute in Philadelphia.

Signed F. B. CROCKER,
W. E. GEYER.
G. A. HAMILTON,
A. E. KENNELLY, Chairman
W. D. WEAVER.

A paper read at the 17th General Meeting of the American Institute of Electrical Engineers, Philadelphia, May 16, 1900, President Hering in the Chair.

A PERCENTAGE BRIDGE.

BY HERSCHEL C. PARKER.

There are many cases in the measurement of resistance that require the exact determination of comparatively small differences. This is true when standard ohms are compared, when a standard rheostat is calibrated by comparing directly with a standard ohm, in the measurement of temperature coefficients, etc.

For the comparison of standard resistances of the same value, the Carey-Foster method, using the "commutator bridge," furnishes a very precise and satisfactory means of measurement.

If, however, a rheostat in which the coils are arranged in sets of 1, 2, 3, 4 or 1, 2, 2, 5 is to be calibrated against a standard ohm, the problem becomes an exceedingly difficult one. In fact two rheostats have to be calibrated at the same time and the corresponding calculations are very involved.

The method known as "substitution in the bridge" (described in "A Systematic Treatise on Electrical Measurements") is perfectly adapted to the adjustment of the coils of a standard rheostat of any pattern whatever by comparison with a standard ohm, but requires the use of interpolation resistances and the observation of galvanometer deflections when a table of corrections is to be made out for a rheostat already adjusted.

The method devised by the writer, which may be called the "percentage method" and the apparatus employed the "Percentage Bridge," while based on this method, differs from it radically, it is a zero method, makes use of readings on the bridge wire in place of galvanometer deflections, and does away entirely with the use of interpolation resistances.

It seems to possess all the advantages of the Carey-Foster method, while the apparatus required is much simpler using only

4 mercury cups in place of the 20 mercury cups of the complicated commutating device usually made use of. At the same time it gives the great range of measurement of the method of percentage differences.

The arrangement of the apparatus is shown in Fig. 1.

The "percentage coils" $c\,c$ are joined to the ends of the bridge wire $A\,B$ and these coils are so adjusted that when resistance in one arm of the bridge is changed by an amount θ the displacement on the bridge wire $= \lambda$.

Let $L = 1$ ohm and $R = 1$ ohm.

Then $(C + A) - \lambda : (C + B) + \lambda : : L : R + \theta$.

Now when $\theta = .01$ ohm, adjust the coils $c\,c$ so that $\lambda = 100$ divisions of the bridge wire, then 1 division $= .01\%$ (approximately).

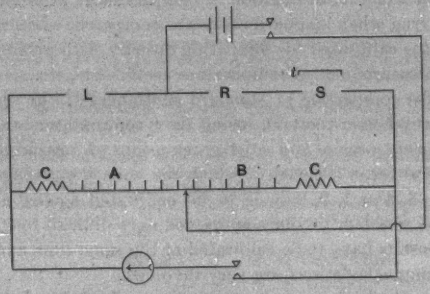


Fig. 1.

If $L = 10$ ohms and $R = 10$ ohms, when $\lambda = 100$ divisions $\theta = 0.1$ ohm (1% of 10 ohms) and 1 division is again .01%.

This being true for any values of L and R, the bridge when once adjusted will always read in percentage.

The value of the "percentage coils" for any given value of the wire AB can be calculated as follows:

Let $A + B = 1$ ohm $= 1000$ scale divisions, and the ratio of the change of resistance $= 100/K$.

Suppose the position of balance to be at the center of the bridge wire, then $A = B$ and

$$A + C = B + C = X. \qquad \text{Whence } \frac{X - \lambda}{X + \lambda} = \frac{100}{K},$$

$$KX - 100X = K\lambda + 100\lambda, \qquad X = \frac{\lambda(K + 100)}{K - 100}.$$

If $K = 101$ and $\lambda = 0.1$ ohm (100 divisions),

$$X = \frac{0.1\,(101 + 100)}{101 - 100} = 20.1 \text{ and } C = X - A = 20.1 - .5 =$$

19.6 ohms.

To find how nearly this value of C will give the true difference in percentage for any other value of λ we may substitute in the equation $\frac{X-\lambda}{X+\lambda} = \frac{100}{K}$, thus when $\lambda = .001$ ohm (1 division) and K should equal 100.01, $\frac{20.1 - .001}{20.1 + .001} = \frac{100}{K}$, $K =$ 100.00995, which gives an error of only .00005%, and when $\lambda = .01$ ohm (10 divisions), $K = 100.09955$, an error of only .00045%. These errors are, of course, quite beyond the errors of observation:

Thus we may have a bridge that will read in differences of percentage with the greatest accuracy from considerably above 1% to differences in resistance as small as .001%.

By taking different values for λ and K in the equation $X = \frac{\lambda\,(K + 100)}{K - 100}$, and then calculating from the values obtained for X the corresponding values for K from the equation $\frac{X-\lambda}{X+\lambda} = \frac{100}{K}$, it is found that the best ratio of the bridge wire to the "percentage" coils for all usual cases of measurement is 1 : 19.6.

If it is desired to have 1 division of the bridge wire = .001%, then the ratio should be, 1 : 200.5.

If the point of balance is not exactly at the center of the bridge wire the error introduced is very small. Thus, suppose in the case where 1 division = .01% the position of balance was 100 divisions (0.1 ohm) from the center, then when balanced the ratio is, 20.2 : 20.0 : : 1.01 : 1, and when $\lambda = 100$ divisions (0.1 ohm) the ratio is 20.3 : 19.9 : : 1.0201 : 1. Now 1.0201 — 1.0100 = .0101, an error of only .01%.

This error may, of course, be eliminated if necessary by adding compensating wires to either arm of the bridge when the point of balance is a considerable distance from the center.

This form of bridge is of great service in the laboratory when exact and rapid comparisons are desired. An example will best show the method of employment.

Queen Rheostat No. 1 compared with *Standard Ohm* = 1.0005 at 19° C.

Positions on Balance.	Bridge Wire Substitution.	Difference in %.	Correct Value.	No. of Coil.
500	495	— .05	1.0000	1
436	435	+ .09	2.0023	2
409	395	— .14	2.9986	3

This method is especially adapted to the determination of temperature coefficients, for the change from the position of balance divided by the difference in temperature gives at once the coefficient. Since a change in resistance of .001% can easily be detected, the temperature coefficient when a difference of temperature of only 1 degree is employed can be accurately measured. This bridge also gives a convenient means for determining small changes in temperature. If a wire whose temperature coefficient = 0.2% is balanced in one arm of the bridge, then it is possible to measure a change of temperature = .005 degree.

The "Percentage Bridge" is so easy to construct and adjust, being, in fact, only an ordinary bridge wire with the addition of the "percentage" coils, that it seems possible it may be found useful in many electrical laboratories when the precise comparison of standard ohms or the calibration of standard rheostats is required.

A paper presented at the 17th General Meeting of the American Institute of Electrical Engineers, Philadelphia, May 17, 1900. President Hering in the Chair.

EDDY CURRENT LOSSES IN TRANSFORMERS.

BY FITZHUGH TOWNSEND.

Experiments dealing with the peculiar phenomena connected with the magnetization of iron have been in progress at Columbia University for a number of years.

In the spring of 1899, a further attempt was made to study the components of the exciting current of a transformer, and their relation to the fundamental component, together with the laws governing the phase displacement and amplitude of the latter. This investigation is still going on, and on completion, the results will be published by Dr. Pupin in conjunction with the author of the present paper.

The experiments described in these pages are an offshoot of the main research just alluded to. Their object is to show that the eddy current loss in a transformer appears to fulfil the relation

$$W_e = K B^{1.6} f^2 \text{ and not}$$

$$W_e = K B^2 f^2$$

as is generally assumed to be the case.

It was found in 1899 that the phase of the exciting current of a transformer was practically independent of the frequency, and varied with the flux density.

A coil wound upon an iron core does not behave like an ordinary induction. The current has a certain hysteretic angle of lag which depends only on the magnetic induction. This angle of lag is probably due to some magnetic property of iron which is not as yet understood. It is not due to a true inductance.

According to the experiments to be described, the equation

$$W = B^{1.6}(K_1 f + K_2 f^2)$$

represents the hysteresis and eddy current losses in a transformer.

$$K_1 B^{1.6} f = W_h = \text{hysteresis loss},$$
$$K_2 B^{1.6} f^2 = W_e = \text{eddy current loss}.$$

The modification of the generally accepted expression for the core losses of a transformer involved in this equation, lies in the fact that the eddy current losses are proportional, not to the square, but to the 1.6th power of the magnetic induction.

If, as appears to be the case, the eddy current losses do vary in this manner, it is a fact so peculiar, and so opposed to preconceived ideas on the subject, that some attempt at explanation is necessary.

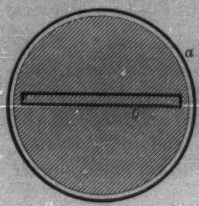


Fig. 1.

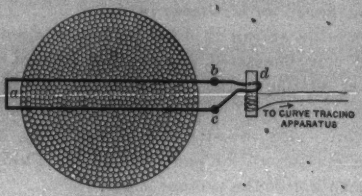


Fig. 2.

In order to arrive at an explanation, the most promising method of procedure will be to consider carefully the conditions under which eddy currents occur.

Fig. 1 represents a section of the core of a transformer. a is a short-circuited turn wound around the core. b is likewise a short-circuited turn, the only difference being that it is embedded in the iron. If the core is laminated horizontally, b will represent substantially the path followed by the eddy currents in a single sheet of the laminations.

The iron core is supposed to be uniformly wound, so as to form a perfect solenoid. Let the ohmic resistance of this winding be so small as to be negligible. A simple harmonic electromotive force is impressed on the magnetizing turns. Let us assume that there are no eddy currents throughout the mass of the iron, and that the induction is uniform. It will also be of

necessity a simple harmonic function, that being the form of the impressed electromotive force.

Owing to the varying permeability of the iron, however, the magnetomotive force H will be a complex harmonic.

The turn b embedded in the iron encloses so few lines of force that the coefficient of mutual induction between b and the magnetizing turns is very small. On the other hand, all the lines of induction which pass through b are likewise interlinked with the magnetizing turns.

If now a simple harmonic electromotive force is impressed upon the primary circuit, the secondary b will have induced in it a similar electromotive force. This secondary E. M. F. will be 180° behind the primary E.M.F.

The resulting secondary current c_b will produce no appreciable effect on the primary current, either in respect to phase or amplitude. The reason for this is that the electromotive force of mutual inductance in the primary circuit due to the current in the coil b, namely c_b, is negligible.

c_b, however, has an effect on the magnetic distribution. It is a counter magnetomotive force, and it must combine with the primary magnetomotive force H to produce a simple harmonic flux.

The current c_b, therefore, is a magnetomotive force acting in such a way as to give rise to a simple harmonic flux in the iron. It is, therefore, to be expected that it should possess, in common with the primary current, those peculiar characteristics with regard to wave shape and phase displacement which characterize every magnetomotive force producing a simple harmonic flux in iron.

The current c_b should lag behind the secondary electromotive force to the same extent that the primary current lags behind the primary electromotive force. This angle of lag depending only on the magnetic induction and not on the frequency, as already stated. c_b should also be a complex harmonic current wave, similar in shape and following the same laws as those which govern the primary current.

When the flux density varies, the electromotive force acting on b increases in direct proportionality as in the case of the primary circuit; the frequency remaining constant. Now, if the current c_b follows the same laws with regard to phase displacement and wave-shape, as those which govern the primary mag-

netomotive force H, then the energy in b must vary as the 1.6th power of the induction as is the case in the primary of the transformer described in connection with Fig. 1.

If the frequency varies while the induction remains constant, then, under the supposition of similarity between c_b and the magneto-motive force H, the phase displacement of c_b should remain constant. Therefore, since the voltage and current in the circuit b are each directly proportional to the frequency, the watts in b must vary as the square of the frequency.

In order to pass from Fig. 1 to the case of an actual transformer, we have only to perform an integration by considering the effect of a large number of circuits similar to b lying in planes parallel to each other. The integral energy loss, repre-

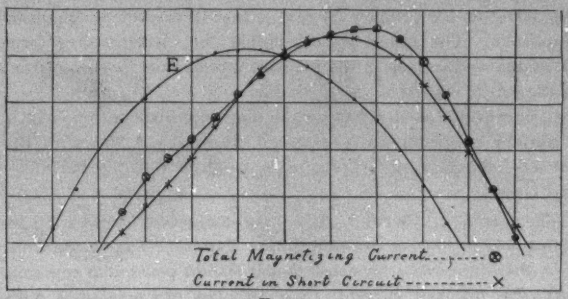


FIG. 3.

senting as it does, the eddy current loss in a real transformer, may be expressed by the equation.

$$W_e = K B^{1.6} f^2.$$

It is interesting to compare the conditions which exist in a circuit such as a in Fig. 1, with those just discussed as characteristic of the circuit b.

a as has been already explained, consists in a turn of wire forming a short-circuit around the core.

The mutual inductance between a and the primary turns is not negligible. Also, a will have considerable inductance, since a current flowing in a will produce a leakage field, the lines of force of which will not cut the primary turns.

A current in a can only affect the magnetic distribution in so

far as it blows out some of the lines of force. The path of these lines is mainly through air and not through iron, hence the magneto-motive force, to which they are due, should not be expected to be a complex-harmonic wave. The current in a should, however, lag behind its electromotive force, owing to the inductance of the circuit a.

In order to discover if there is any foundation for the above speculations, a core made of iron wire was procured, and some magnetizing turns wound uniformly around it. Two brass plates were stuck through the body of the core and soldered together at a as shown in Fig. 2. The two other ends b and c were connected together by a short and heavy copper cable.

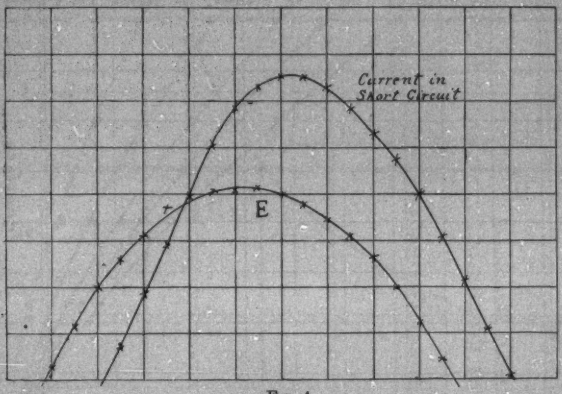


FIG. 4.

One turn of this cable was put around the small open circuit transformer d, and the curve of the current in the short-circuited turn $a\ b\ c$ obtained[1].

The circuit $a\ b\ c$ is a fair imitation of the elementary current b of Fig. 1. The magnetization of the transformer d is so small as not to interfere materially with the results.

Fig. 3 shows curves of impressed primary electromotive force and primary current, in addition to the curve of current in the short circuit a, b, c. It will be seen that this latter appears as a lagging complex harmonic wave, as suggested above.

1. The method by which the curve could thus be determined is described in TRANSACTIONS, Vol. XVII. No. 1.

Fig. 4 shows two curves. The one marked E is the impressed electromotive force curve. The other is the curve of the current in a short-circuited turn of wire entirely encircling the core. This curve is a practical example of the current in coil *a*

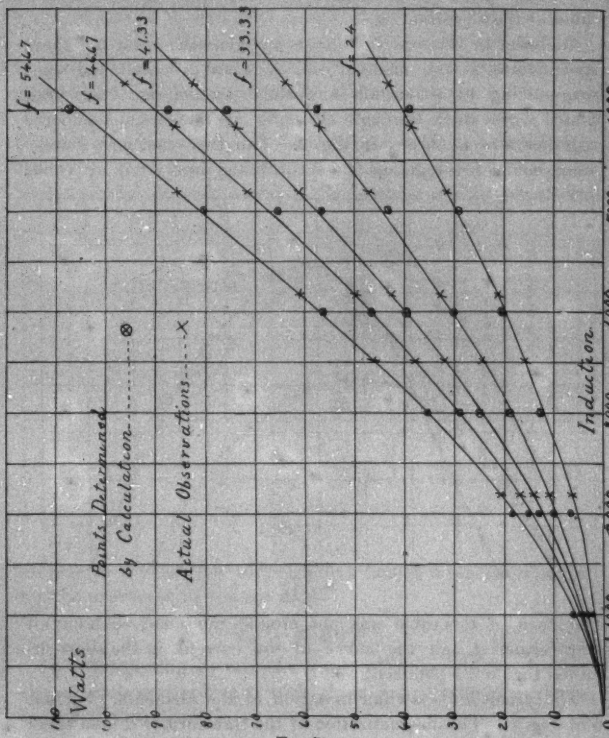


FIG. 5.

as discussed in connection with Fig. 1. It is very nearly a simple harmonic, and lags behind the impressed electromotive force.

An important corollary follows from the expression

$$W = B^{1.6}(K_1 f + K_2 f^2)$$

if this equation is indeed the correct one. This corollary is that

the ratio of hysteresis to eddy current losses is independent of the magnetization. Let Wh and We represent these losses, then

$$\frac{W_h}{W_e} = \frac{K_1}{K_2 f}$$

Also, the relation between $\frac{W_h}{W_e}$ and f is an equilateral hyperbole. This relation is what might

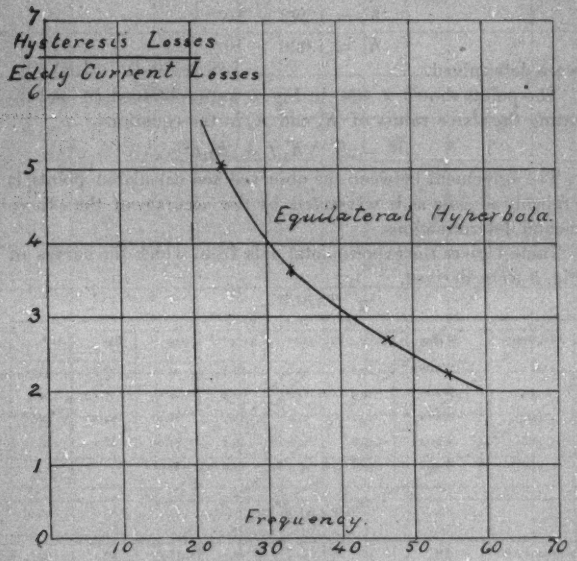


FIG. 6.

be expected, as there is no doubt that

$$\frac{W_h}{W_e} = \infty \text{ when } f = 0,$$

and probably

$$\frac{W_h}{W_e} = 0$$

at very high frequencies.

Prof. W. Peukert, in the *Elektrotechnische Zeitschrift* of

September 21st, 1899, published an article on the iron losses of a transformer, in which a large number of measurements of core losses at different frequencies and magnetizations are tabulated.

The transformer used in his experiments was a closed circuit laminated iron ring built of sheets 19.7 mils in thickness.

These results were plotted as shown in Fig. 5. By taking a number of simultaneous values, the average value of the constants

$$K_1 = 1.223 \times 10^{-6}$$
$$K_2 = 1.006 \times 10^{-8}$$

were determined.

The points shown ⊕ ⊕ ⊕ in Fig. 5 were obtained by substituting the above values of K_1 and K_2 in the equation

$$W = B^{1.6}(K_1 f + K_2 f^2)$$

The agreement between the observed and calculated points is probably as close as is warranted by the accuracy of the experimental determinations.

Table I gives the experimental data from which the curves of Fig. 5 were derived.

TABLE I.

Frequency.	B max.	2187	3522	4173	5192	5843	6260
24	Watts	6.2	13.9	21.3	31.4	38.5	43.2
33.33	Watts	10.9	24.7	34.9	47.0	57.1	64.0
41.33	Watts	14.1	32.2	42.7	61.2	74.0	82.6
46.67	Watts	16.8	37.8	49.8	71.4	86.2	96.3
51.67	Watts	20.8	46.3	60.9	85.7		

TABLE II.

Frequency.	αB	30	39.3	48.2	58.5	66.5	74.5
49.7	Watts	26.3	40.5	54.6	74	91	112
70.2	Watts	35	58	79	101	127	152
90.7	Watts	50	76	105	136	171	208
121.7	Watts	70	105	144	192	235	284
134.3	Watts	76	114	161	213	263	314

Fig. 6 shows the equilateral hyperbola relating $\left(\frac{W_h}{W_e}\right)$ and f, for the data given by Prof. Peukert.

The experiments just mentioned were made, it must be re-

membered, on a transformer with closed magnetic circuit. It seemed desirable, therefore, to obtain similar data from a commercial transformer.

The results in Table II were obtained from a five kilowatt

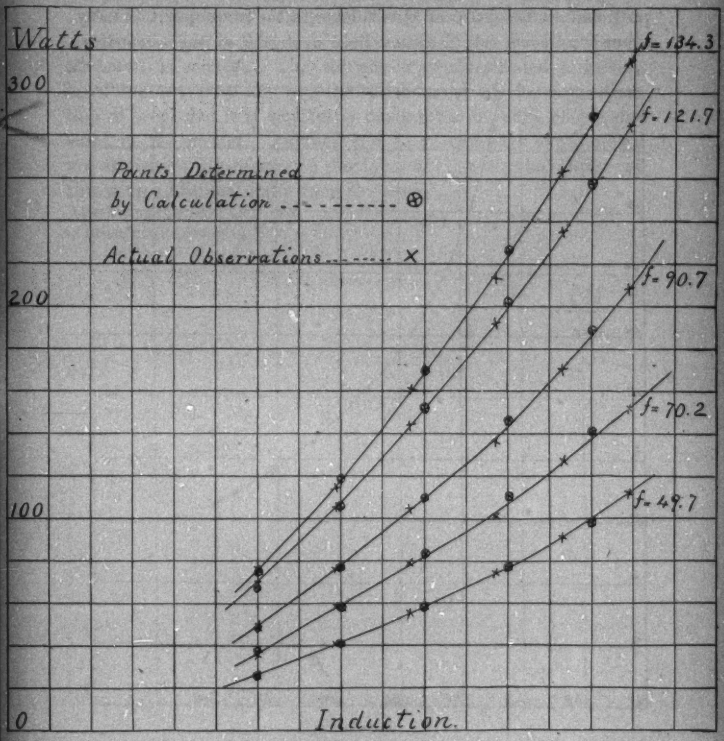


FIG. 7.

Stanley transformer. Inasmuch as neither the cross-section of the iron nor the number of turns were known, it was not possible to determine the magnetic induction. The core loss in watts was therefore measured at different frequencies, for six different field strengths of the alternator used to excite the transformer.

The numbers in the column headed α_B are therefore not flux densities, but the voltages at the different field strengths at the frequency of 121.7. These voltages are proportional to the magnetic induction in the transformer at the different field strengths of the alternator, since the magnetic induction is proportional to the ratio of the voltage to the frequency. At any other frequency, for the same field strengths of the alternator,

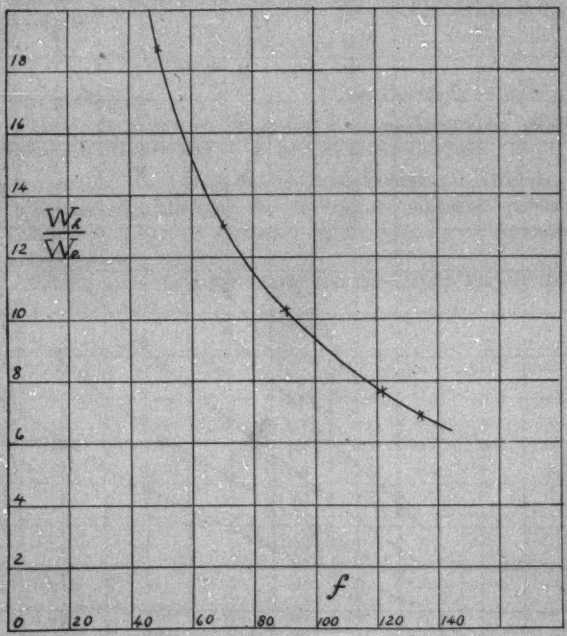


FIG. 8.

the corresponding values of the induction will be necessarily unchanged, since the ratios of the impressed voltages to the frequency are the same as at the former frequency for each and every field strength.

The data of Table II is plotted in Fig. 7. The formula for the core losses was then applied in the same manner as in the

case of Fig. 5. The agreement between the calculated values and those determined by experiment is again quite satisfactory. Fig. 8 shows the equilateral hyperbola, giving the relation between $\left(\frac{W_h}{W_e}\right)$ and f.

The close agreement between theory and experiment in the results described in this paper go far to indicate that the theory advanced is correct. The subject is so complicated, however, involving as it does the peculiar phenomena of the magnetization of iron, that it is manifestly dangerous to jump at conclusions in the matter. Further and more complete experiments are at present in progress in this laboratory, with the object of throwing additional light upon the subject.

Alternating Current Laboratory,
Electrical Engineering Dept.,
Columbia University,
April, 1900.

APPLICATIONS FOR ASSOCIATE MEMBERSHIP.

Applications have been received from the following candidates for Associate Membership, which will be acted upon by Council at its meeting, July 23, 1900.

1679. E. D. Carnaghan, Durango, Do, Mexico.
1680. James Meikle Eglin, Philadelphia, Pa.
1681. Dudley Farrand, Newark, N. J.
1682. Wm. Stuart Smith, Berkeley, Cala.
1683. Albert P. Allen, New York City.
1684. Walter F. Smith, Philadelphia, Pa.
1685. Roderick J. Parke, Toronto, Canada.
1686. L. E. Imlay, Niagara Falls, N. Y.
1687. George Nathan Tidd, Chester, Pa.
1688. Arthur O. Baker, Wichita, Kansas.
1689. James Martin Kent, Kansas City, Mo.
1690. Frank H. Varney, San Francisco, Cala.
1691. Samuel L. Nicholson, New York, N. Y.
1692. Bret Harter, Rochester, Mich.
1693. Jeremiah J. Kennedy, New York, N. Y.
1694. F. P. Spiese, Tamaqua, Pa.
1695. Frederick B. Smith, Boston, Mass.
1696. Lorenz D. Hitzeroth, San Francisco, Cala.
1697. George H. Berg, Boston, Mass.

Any member objecting to the election of any of the above candidates should notify the Secretary to that effect prior to July 23, 1900.

A paper read at the 17th General Meeting of the American Institute of Electrical Engineers, Philadelphia, May 19th, 1900, President Hering in the Chair.

WAVE TRANSMISSION OVER NON-UNIFORM CABLES AND LONG-DISTANCE AIR-LINES.

BY M. I. PUPIN, PH. D.

This paper describes an experimental investigation of a method of constructing cables and long-distance air-lines for power transmission by electrical waves, particularly for long-distance telephony and telegraphy. This method is a practical application which offers an experimental test of the general mathematical theory of wave-propagation over non-uniform conductors, which is given in the second part of the paper. The first part contains a physical explanation of this mathematical theory and a description of experimental researches bearing upon it. It also describes an experimental verification of long-distance telephony over non-uniform cables.

Physical Theory of Electrical Wave-Propagation over Cables and Long-Distance Air-Lines.

A. *Wave Propagation Over Uniform Conductors.*—Transmission of electrical energy over conducting wires is a *wave transmission* when the distance between the transmitting and the receiving apparatus is sufficiently long to permit the development of electrical waves. Such a transmission exists in long-distance telegraphy and telephony. It does not exist to any practically appreciable extent in ordinary transmissions of electrical power by alternating currents over distances which up to the present time have been bridged over by electrical power transmission lines. In cases of wave transmission considered here the conductors will be called *wave conductors.* The circumstances attending wave transmission are considerably different from those attending ordinary electrical transmission and should be carefully differentiated from them. In ordinary transmission

the reactions set up in the receiving apparatus are the most essential reactions which the force impressed by the transmitting generator has to overcome. The reactions set up in the transmitting line itself are small in comparison to it. The case is analogous to the transmission of power from the piston of a steam engine to a motor connected to the engine by a short, stiff, piston rod. The reactions set up in the piston rod itself are small in comparison to the reactions which the motor opposes to the driving pressure. Hence, neither the elastic nor the kinetic reactions of the piston rod, nor the reactions due to internal frictional resistances in the rod are seriously thought of when we analyze the reactions attending this case of power transmission. But consider now what will happen if we increase the distance between the piston and the receiving motor and consequently increase the length of the piston rod. We can no longer consider the rod as a perfectly rigid connection between the driving pressure of the piston and the reactions of the receiving motor. The rate at which the piston delivers energy at any moment is not equal to the rate at which energy is delivered at the receiving motor at that moment. There is a lag in phase. The energy transmitted is first stored up in the piston rod and then delivered from the rod to the receiving motor. While it is stored up in the rod it exists partly as kinetic energy of the moving mass of the rod and partly as potential energy, due to the rod's elastic deformations. The process of transmission consists in successive transformations of the kinetic into the potential energy of the rod and vice versa. These transformations being progressive the energy is propagated along the rod, and we say that the propagation is a wave propagation, in order to state in a single word that the progressive motion along the rod is a periodic one. Analogous conditions exist when electrical energy is transmitted by a periodically varying electromotive force acting on a long conductor. The transmission is not a direct one; the transmitted energy is first stored up in the medium surrounding the transmission line, and from there it is transferred to the receiving apparatus. While it is stored up in the medium it exists there partly as magnetic energy stored up in the field of magnetic flux and partly as electrical energy stored up in the field of electrical flux. The process of propagation consists in the progressive transformation of the magnetic into the electrical energy and vice versa. When the electromotive force impressed by the transmitting generator is a peri-

odic one the propagation will be in the form of electrical waves. The expression "electrical wave" is nothing more nor less than a brief statement of the physical fact that in the case under consideration the energy which at any moment is stored up in the medium surrounding the transmission line is distributed periodically over this line. The current and the potential also vary periodically. At points of maximum magnetic energy the current is maximum, and at points of maximum electrical energy the potential is maximum. Roughly speaking, points of maximum current are points of minimum potential and vice versa.

Wave-length.—Consider now the distance between any two consecutive points of minimum current or minimum potential. This distance is a half wave-length. Suppose that the impressed electromotive force is a simple harmonic of frequency 600 p.p.s. Say that we find the wave-length to be 18 miles, the velocity of propagation will be 10,800 miles per second; considerably less than the velocity of propagation of light through a vacuum. To some this numerical illustration may seem as highly improbable, for we are accustomed to hear much of electricity being propagated with the velocity of light. But it should be remembered that this is true under certain particular conditions only. The velocity of propagation of electrical waves of telephonic frequencies over conducting wires may be anything from the velocity of light down to a few inches, or even less than an inch, per second, depending on the inductance, resistance, and capacity of the line. The smaller the velocity the shorter, of course, will be the wave-length for a given frequency. *The wave-length*, as will be seen presently, plays a very important part in this investigation. It is considered here as one of the characteristic constants of wave propagation. I am not aware that previous investigators of the propagation of long electrical waves have devoted any serious attention to this characteristic constant.

Attenuation Constant.—There still remains another constant which with the wave-length completely defines electrical wave propagation. It is called here the *attenuation constant*. To bring out its physical meaning consider two consecutive half wave-lengths at any moment. The one nearest to the transmitting apparatus shall be denoted by A and the other by B. The wave energy stored up in the medium surrounding A is greater than that stored up in the medium surrounding B. Hence wave energy is gradually dissipated during its propagation from the

transmitting to the receiving apparatus, and therefore the amplitude of both current and potential become smaller as the energy progresses along the transmission line. Let U be the amplitude of the current at the transmitting end, and U_s be the amplitude at a distance s, then if the line be considered infinitely long

$$\frac{U_s}{U} = e^{-\beta s}$$

where e is the base of Naperian logarithms. The constant β is called here the attenuation constant. The mathematical expression for β is well known

$$\beta = \sqrt{\tfrac{1}{2} p C [\sqrt{p^2 L^2 + R^2} - p L]}$$

where L, R, C, are the inductance, resistance, and capacity, respectively, of the wave conductor per unit length, and p is the frequency speed. Much confusion exists in the minds of physicists as to the real significance of this constant, and as to the true cause of current attenuation. It is usually stated that the capacity of the line, acting as it does somewhat like a shunt, is the cause of all the trouble experienced in electrical wave transmission. This statement contains a small part of the truth, only, and for that reason may and actually has become misleading. The fact that a conductor possesses inductance and capacity shows that the medium surrounding it is capable of storing up energy, which, indeed, is a blessing; it cannot possibly signify that energy propagated along it will be dissipated; and if capacity cannot cause a loss of energy, how can it possibly cause an attenuation of current? The dissipation is due to imperfect conductivity of the wire, and to that alone. Inductance and capacity regulate it, they do not cause it.

Consider now the manner in which this regulation is effected. The dissipation of the energy transmitted occurs at the time when it is stored up in the medium as magnetic energy; for if the medium surrounding an element ds of the transmission wire contains a quantity dW of magnetic energy, then a current x must flow in that element such that

$$dW = \tfrac{1}{2} L x^2 \, ds$$

Let dH be the rate of dissipation in that element, then

$$d\,H = Rx^2\,d\,s$$

Suppose now that by some means we increase L to $n^2\,L$, the medium surrounding the elements ds will store up the same amount dW of magnetic with one n^{th} of the current, for

$$d\,W = \tfrac{1}{2}\,n^2\,L\,(x/n)^2\,d\,s$$

Let dH_1 be the rate of dissipation in this case then

$$d\,H_1 = R\,(x/n)^2\,d\,s = d\,H/n^2$$

It follows, therefore, that during the transmission of a given quantity of energy over a conducting wire the dissipation will be diminished by increasing the inductance of the wire, for if the wire have high inductance then small currents are required to transmit a given quantity of energy, and small currents incur small ohmic resistance losses.

By increasing the inductance the efficiency of transmission is increased just as effectively as by increasing the conductivity of the transmission wire.

Distortionless Wave Conductors.—Another important advantage is gained by increasing the inductance. The expression for β given above shows that attenuation depends on frequency; it increases with it. Hence, in telephonic transmission where waves of complex harmonic frequencies are propagated over the line there will be distortion of the waves, because upper harmonics will be attenuated more vigorously than the lower frequencies. This results in a distortion of speech, which is noticed in long-distance telephonic transmission as defective articulation. Some instructive experiments bearing upon this point will be described presently. High inductance obviates this difficulty. To illustrate—suppose that the inductance is large in comparison to the resistance, the expression for β will reduce to

$$\beta = R/2\sqrt{C/L}.$$

It is independent of the frequency. All frequencies are attenuated alike, so that high inductance not only diminishes attenuation but also renders the circuit distortionless. Such a circuit is the ideal circuit for telephonic and telegraphic wave transmission.

Mr. Oliver Heaviside, of England, to whose profound researches most of the existing mathematical theory of electrical wave propagation is due, was the originator and most ardent advocate of wave conductors of high inductance. His counsel did not seem to prevail as much as it deserved, certainly not in his own country. I trust that the physical view of attenuation described above in the terms of the dissipation of energy which is transmitted over the wire will help to elucidate Mr. Heaviside's theory of high inductance wave conductors.

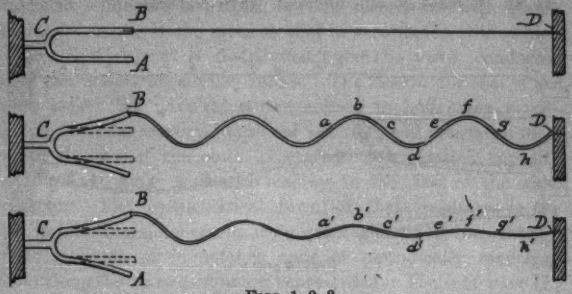
Wave Propagation over Non-Uniform Conductors.—But Mr. Heaviside's proposition to employ wave conductors of high inductance contained a serious difficulty which his mathematical theory was not capable of overcoming. The difficulty is this: How can a wave conductor be constructed so as to have a high inductance? Ordinary circuits can be endowed by as much inductance as may be required by simply introducing a coil of proper dimensions, with or without an iron core, into it. But this will never do in the case of a wave conductor; for a coil introduced that way will act by reflection as a barrier to electrical waves. Acting on a suggestion made by Mr. O. Heaviside[1] in 1893, wave propagation experiments over long wave conductors containing a certain number of coils in series at periodically recurring points have actually been tried by telephone engineers; the results obtained have invariably proved most disappointing. I shall return to this point later on in connection with the description of experiments given further below. Suffice it to state here that all attempts to increase the inductance of a wave conductor by the introduction of inductance coils at periodically recurring points failed, because they had no mathematical theory to guide them, so as to avoid the difficulties of wave reflection by inductance coils introduced that way. It is hardly worth while to enter here into a discussion of the many other attempts at increasing the inductance of a wave conductor by devices which had neither theory nor experiment to recommend them. In fact, most of them were absurd on the face of it.

The first mathematical theory dealing with wave propagation over conductors of this kind was presented by the author before this INSTITUTE on March 22d, 1899.[2] A more complete theory is given in the second part of this paper. The main features of

[1] Heaviside, O.—"Electromagnetic Theory," vol. i., p. 485.

[2] Pupin, M. I.—"Propagation of Long Electrical Waves;" TRANSACTIONS, vol. xvi., p. 93, 1899.

this theory are extremely simple and can be explained by a simple mechanical illustration. Consider the arrangement of Fig. 1. A tuning fork has its handle C rigidly fixed. To one of its prongs is attached a flexible inextensible cord $B\ D$. One terminal of the cord is fixed at D. Let the fork vibrate steadily, the vibration being maintained electromagnetically or otherwise. The motion of the cord will be a wave motion. If the frictional resistances opposing the motion of the cord are negligibly small the wave motion will be approximately that of stationary waves as in Fig. 2. The direct waves coming from the tuning fork and the reflected waves coming from the fixed point D will have nearly equal amplitudes and by their interference form approximately stationary waves. If, however, the frictional



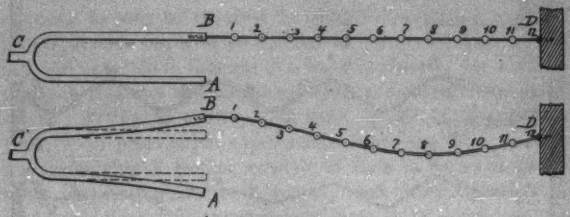
FIGS. 1, 2, 3.

resistances are not negligibly small, then there will be dissipation of the propagated wave energy. Hence the direct and the reflected waves will not have equal amplitudes, and, therefore, their interference will not result in stationary waves. The attenuation of the wave is represented graphically in Fig. 3. Experiments will show that, other things being equal, increased density of the string will diminish attenuation, because a larger mass requires a smaller velocity in order to store up a given quantity of kinetic energy, and smaller velocity brings with it a smaller frictional loss. This is a striking mechanical illustration of a wave conductor of high inductance. It should be observed here that an increase of the density will shorten the wave-length.

Suppose now that we attach a weight, say a ball of beeswax, at the middle point of the string, in order to increase the vibrating mass. This weight will become a source of reflections and

less wave energy will reach the point D than before. The efficiency of transmission will be smaller now than before the weight was attached. Subdivide now the beeswax into three equal parts and place them at three equidistant points along the cord. The efficiency of wave transmission will be better now than it was when all the wax was concentrated at a single point. By subdividing still further the efficiency will be still more improved; but a point is soon reached when further subdivision produces an inappreciable improvement only. This point is reached when the cord thus loaded vibrates very nearly like a uniform cord of the same mass, tension, and frictional resistance. Such a loaded cord with a tuning fork attachment is represented in Fig. 4.

If an increase in efficiency of wave transmission over a cord



FIGS. 4 and 5.

thus loaded is to be obtained, it is evident that the load must be properly subdivided and the fractional parts of the total load must be placed at proper distances apart along the cord, otherwise the detrimental effects due to reflections resulting from the discontinuities thus introduced will more than neutralize the beneficial effects derived from the increased mass.

The problem of finding the proper distance at which the loads should be placed is a definite mathematical problem of Analytical Mechanics, but unfortunately it was never solved. Fig. 5 represents a cord carrying loads at proper distances apart. Experiments with cords of this kind will soon convince one that the distance between the loads should be considerably smaller than one-half of the wave-length of the wave which is to be transmitted. So that though a given cord may be properly loaded for some wave-length it will not be properly loaded for shorter wave-lengths. It is impossible to load a cord in such a way as to make

it equivalent to a uniform cord for all wave-lengths; but if the distribution of the loads satisfies the requirements of a given wave-length it will also satisfy them for all longer wave-lengths. It should be observed now that the wave-length which is considered here is not the wave-length of the cord without the loads, but the wave-length which the frequency under consideration will have on the properly loaded cord, or what is the same thing, on a uniform cord of the same mass, tension, and frictional resistance, as the loaded cord. This point is of fundamental importance, for the wave-length corresponding to a given frequency may and generally will be much shorter on the loaded cord than on the cord without the loads.

A cord of this kind is a mechanical analogy to an electrical wave conductor. The mathematical law in accordance with which such a cord moves is the same as that in accordance with which the electrical current is distributed over the wave conductor under the action of similar forces. The reason for that is not far to seek. We have the same reactions in both cases, viz.:—Kinetic or mass reaction, tensional reaction, and resistance reaction in the case of the cord. Electro-kinetic reaction, capacity reaction, and ohmic resistance reaction in the case of the wave conductor. The mathematical form of these reactions is the same in both cases, hence one is an exact analogy of the other.

The insertion of inductance coils at periodically recurring points along the wave conductor, represented in Fig. 3 of Part II., produces the same effect upon electrical wave transmission as the distribution of the small loads along the stretched cord of Fig. 4 produces upon mechanical wave transmission along the cord. The mathematical theory of wave propagation over non-uniform conductors of this kind is given in Sections I and II of the Second Part of this paper. This theory is, of course, at the same time the mathematical theory of wave propagation over a loaded cord described above. The main object of this theory is find an answer to the question: *Under what conditions are non-uniform conductors described in these two sections equivalent to their corresponding uniform conductors?* The answer which the mathematical theory gives to the question just proposed is definite. To formulate it introduce here a convenient technical term. Consider the distance between two consecutive inductance points, that is, the points at which the inductance coils are introduced. Denote it by l, and let the wave length which

is to be transmitted be λ. Now introduce an angle φ such that

$$\varphi / 2\pi = l / \lambda.$$

The angle φ shall be called the *angular distance* between the inductance points, or inductance sources. The angular distance 2π corresponds to the wave-length. The law which determines the degree of equivalence between a non-uniform conductor and its corresponding uniform conductor can now be stated as follows: **A non-uniform conductor is as nearly equivalent to its corresponding uniform conductor as sin $\varphi/2$ is to $\varphi/2$.**

It is evident that φ is inversely proportional to the wave-length, so that for a given distance between the reactance points the degree of equivalence diminishes as the wave-length diminishes. If a wave of complex harmonic frequency, such as occur in telephony, be transmitted over a non-uniform conductor, then the action of the conductor will be different for the different components of this complex harmonic wave. If, however, the non-uniform conductor acts with sufficient approximation as a uniform conductor toward the highest important frequency of this complex wave then its approximation to a uniform conductor will be even higher for the lower frequencies and thus for all the frequencies of the wave. A numerical example will illustrate this point more clearly. It is well, however, to state more fully the meaning of the expression "equivalence between a non-uniform conductor and its corresponding uniform conductor." All that we can predicate of a wave of a given frequency is that it has a certain wave-length and a certain attenuation constant. Hence, if a wave of a given frequency has the same wave-length and the same attenuation constant on a non-uniform conductor as it has on the corresponding uniform conductor then the two conductors are equivalent to each other. If these two quantities differ by, say, three per cent., then an approximate equivalence up to within three per cent. exists.

Consider now the following numerical example: A twin conductor, such as employed for telephone cables, has a length of 250 miles. Let its constants have the following values per mile:

Inductance = 0.
Resistance = 9 ohms.
Mutual capacity = .074 Microfarads.

According to the high but definite standard which the New

York Telephone Company employs the limiting-distance of telephony over such a cable is 39 miles. According to the lower standard which is maintained in long-distance telephone work the limit would be 78 miles. The results obtained by me experimentally seem to verify these figures. I find that at a distance of one hundred miles telephony over such a cable is very poor, in fact, impracticable, and at a distance of 125 miles impossible. It is proposed now to decrease attenuation and distortion over such a cable by the insertion of inductance coils at periodically recurring points. The attenuation constant β in this case is given by the formula[1]

$$\beta = \frac{R}{\sqrt{2}}\sqrt{\frac{C}{L}}.$$

Say that it is required to have $\beta = .015$. Assume that the introduction of the inductance coils adds 9 ohms per mile, so that $R = 8$ ohms. These values of R and C require $L = .056$ henrys. The attenuation factor at a distance of 250 miles would be $e^{-250\beta} = \frac{1}{40}$, (roughly). That is, $2\frac{1}{2}$ % of the current leaving the transmitting end will reach the receiving end. This is quite sufficient for telephonic purposes; but it should be observed that better efficiency of transmission could be obtained by making L larger. Notice now that the attenuation factor for this cable without the coils would be, for a frequency of 600 p.p.s., roughly, $\frac{1}{25 \times 10^4}$ that is, with a given initial current, the current at the receiving end would be 6000 times larger with the coils than without them.

The next step is to find the wave-length for the highest important frequency in telephony over a *uniform wave conductor* having $L = .056$ henrys, $R = 18$ ohms, $C = .074$ microfarads. The best telephone practice assumes that 750 P. P. S. is the highest frequency of any importance. The wave-length corresponding to this frequency over a *uniform conductor* of this description is obtained from the formula[1]

$$\lambda = \frac{2\pi}{p\sqrt{2LC}} = 14.6 \text{ miles},$$

approximately.

1 See Equation (4a), Sect. I., Part II.

Suppose now that at each mile we place a coil of inductance $L = .056$ henry and a resistance $R = 9$ ohms. The angular distance φ for frequency 750 P. P. S. of the non-uniform conductor thus obtained will be $2\pi / 14.6$ The degree of equivalence of this non-uniform conductor to its corresponding uniform conductor is measured by the degree of equivalence of $\sin \pi/14.6$ and $\pi / 14.6$. Now $\sin \pi / 14.6$ differs from $\pi / 14.6$ by less than one per cent. of the value of $\pi / 14.6$; hence for a frequency of 750 P. P. S. the wave-length and the attenuation constant on the non-uniform conductor will differ from the wave-length and attenuation constant on the corresponding uniform conductor by less than one per cent. of the values of these constants. Such a difference cannot be detected by any of the experimental methods which are at present available for investigating wave propagation. In telephonic transmission the ear could not detect it. For lower frequencies the differences will be even considerably smaller. Hence, the non-uniform conductor thus obtained will represent a uniform, non-attenuating, distortionless conductor for telephonic transmission.

It should be observed here that in the case of a submarine cable of say 2,000 miles the attenuation constant should be much smaller than the value of β given above, in order to have a sufficiently small attenuation factor. Now the capacity per mile of a submarine cable is about four times as large as the capacity of the telephone cable just described. Hence, both on account of the long distance and also on account of the much increased capacity, the inductance per mile will have to be much larger than in the case just discussed. But high inductance and large capacity will give a very short wave-length. For instance, if in the case of the submarine cable having six times the capacity we employ an inductance six times as large as in the case of the telephone cable we shall obtain for the frequency of 750 P.P.S. a wave-length of only $14.6 \div 6 = 2.43$ miles. Hence, since the inductance coils will have to be placed apart at one-sixth of the distance employed in the case of the telephone cable, the distance between them will be about 880 feet. The distance between the inductance coils depends entirely on the circumstances of each particular case. But in all cases the rule given above contains the necessary and sufficient directions.

This rule and the method based upon it are recommended by the mathematical theory developed in Part II of this paper for the construction of distortionless wave conductors of high efficiency

of transmission. It will be shown now in how far this theory is confirmed by experiment.

Experimental Part.

a. Description of the Experimental Cable.—A so-called artificial cable possessing the above constants was constructed by me for the purpose of testing experimentally the mathematical theory developed in Section III of Part II. Previous experiments in this line will be found in the author's paper of March 22d, 1899, cited above.

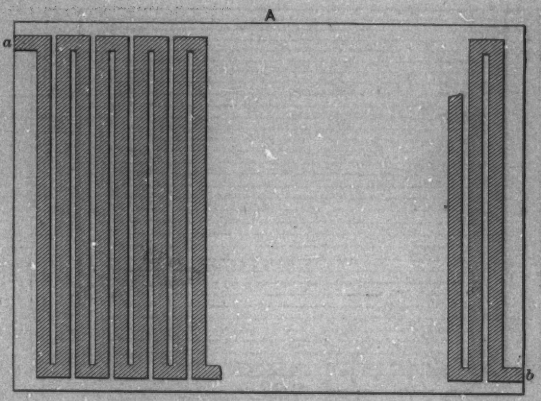


Fig. 6.

The cable has 250 sections. One of these sections is represented in Fig. 6. It consists of a sheet A of paraffined paper. On each side of this sheet is a strip of tinfoil $a\,b$. The resistance of this strip is approximately 9 ohms. The capacity of the condenser formed by the two strips is .074 microfarads approximately. 250 sections of this kind connected in series represent a cable of 250 miles in length having a resistance of 9 ohms and a capacity of .074 microfarads per mile. That such is actually the case was verified experimentally, as will be shown presently. In Fig. 8 the straight lines $A\ B\ C\ D\ A_1\ B_1\ C_1\ D_1$ represnt these sections. The upper lines represent one side of the cable, and the lower lines represent the other side. The sections can be con-

nected in series by couplers to which these sections are attached. One of these couplers is represented in Fig. 9. A wooden strip *A B* has holes drilled through it at regular intervals. Through each hole passes a bolt *c*, the diameter of which is smaller than the diameter of the hole. Two hard rubber washers *a* and *b* hold the bolt in place and prevent it from touching the wood. A nut holds the brass plates *d* in place. Two consecutive brass plates *d d* can be conductively connected by the plug *e*. Screws, as indi-

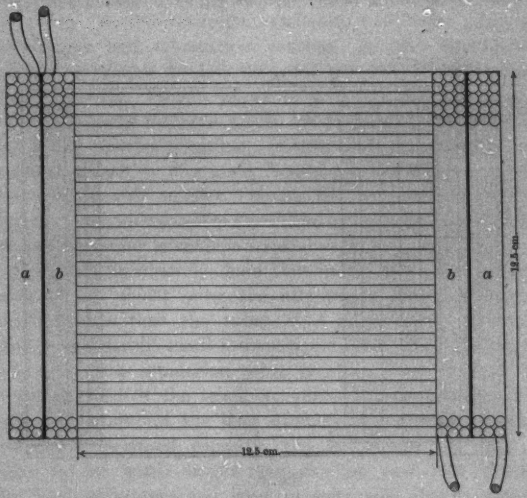


FIG. 7.

cated, connect the terminals *f f* of the condenser sections and of the coils *g g* to the brass plates. These brass plates are indicated in Fig. 8 by numerals 1, 2, 3, ... When the plugs are out, then the consecutive sections of the cable are connected to each other by the inserted coils (see Fig. 8). The plugs being in the coils are short-circuited, and the consecutive sections of the cable are connected to each other directly by the plugs. In this case we have a uniform cable; in the former case we have a non-uniform cable of 250 miles length, with inductance coils inserted at each mile. The coils are wound as represented in Fig. 7. The twin coils aa_1, bb_1, cc_1 of Fig. 8 are represented in Fig. 7 by coils *aa* and *bb*.

These two coils are wound on one spool, and are separated by a sheet of cardboard indicated by a black line in Fig. 7 which is 1-64th of an inch thick. The coil when finished is boiled in beeswax at a temperature of about 280° Fahrenheit, in order to drive all the moisture out and insure good insulation. The dimensions of the coil are given in Fig. 7. Each coil had 580 turns of No. 20 wire B. & S. The average inductance of one of these coils is .030 henry, and the mutual inductance .028 henry. Each coil, therefore, when connected into the line, as indicated in Fig. 8, has an effective inductance of

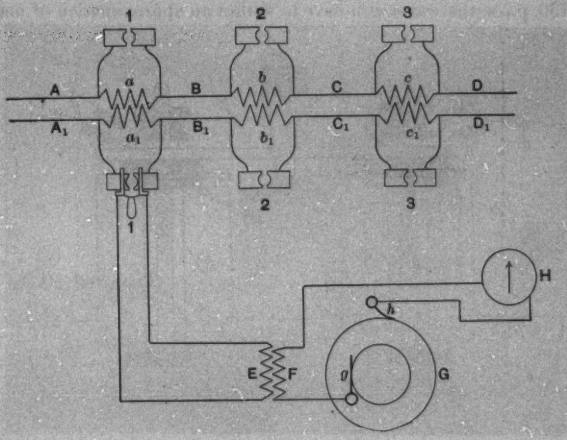


FIG. 8.

.058 henry. This method of bringing the corresponding coils of the two sides of the cable into close relation by mutual induction is not necessary; it is a matter of convenience and economy.

The cable sections are divided in five groups of 50 each, and each of these groups is enclosed in a separate box. Each box represents, therefore, 50 miles of the cable. Fig. 10a is a reproduction from the photograph of one of these boxes. The box marked A is the cable box. On two opposite sides are five rows B....of binding posts, each row having 10 pairs. They

are the terminals of the condenser sections. From these binding posts double cotton covered wires which have been boiled in beeswax lead to the row of brass plate couplers (Fig. 9) indicated in the picture by letters $a, b, c, \ldots$. These boxes were made for me by the well-known mechanician, Mr. E. V. Baillard, to whom my sincere thanks are due for the keen interest which he took in this exceedingly laborious work. The rest of the work on the cable was done in the Columbia University laboratory for electromechanics. The insulation per section was about 50 megohms.

b.—Theory of the Experimental Method.—The mathematical theory of Section III, Part II, states that up to a frequency of 750 p.p.s. the waves will have to within an approximation of one

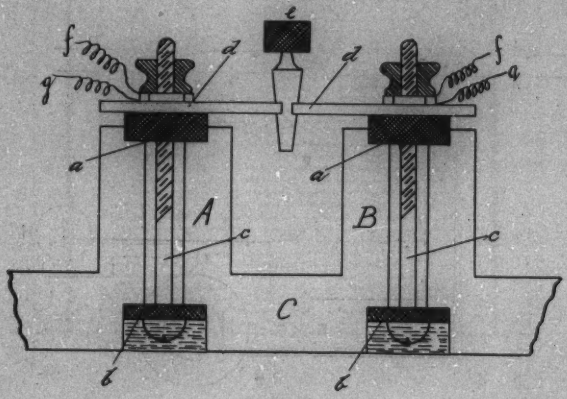


FIG. 9.

per cent. the same wave-length and the same attenuation constant on the cable just described as on its corresponding uniform conductor. This is the side from which experiment has to approach the theory. The method employed in this investigation has, therefore, for its object the determination of wave-length and attenuation constant. Its theory can be stated as follows:

The current η at any point ξ of a uniform loop can be expressed.[1]

1. M. I. Pupin:—Propagation of Long Electrical Waves, equ. (7); TRANSACTIONS, vol. xvi., p. 93, 1899.

$$\eta = A \left\{ e^{\beta\xi} \sin (pt - a\xi) + e^{-\beta\xi} \sin (pt + a\xi) \right\}$$

Let $M(\eta)$ denote the mean value of the current between $t = t_1$ and $t = t_1 + T/2$, then

$$M(\eta) = \frac{y_0}{2} \left(e^{\beta\xi} + e^{-\beta\xi} \right) \cos (pt_1 - a\xi)$$

Here ξ is the distance from the middle point of the loop. The time is counted from the moment when η is zero at the

FIG. 10 A.

middle point. If we then determine the mean values from the moment when η vanishes at the middle point we shall have $t_1 = 0$ and

$$M(\eta) = \frac{y_0}{2} \left(e^{\beta\xi} + e^{-\beta\xi} \right) \cos a\xi$$

or since $\lambda = 2\pi/a$, where λ is the wave-length

$$M(\eta) = \frac{y_0}{2} \left(e^{\beta\xi} + e^{-\beta\xi} \right) \cos \frac{2\pi}{\lambda} \xi \qquad (1)$$

y_0 is evidently the mean value of the current at the middle point. This formula forms the basis of the experimental method employed here. From it the wave-length and the attenuation constant were determined.

c. Determination of the Wave-length.—1. To investigate the wave-length and the attenuation constant of a given frequency it was necessary to impress a simple harmonic electromotive force upon the cable. This was obtained as follows:— A small alternator of thirty poles running at a normal speed of 2400 revolutions per minute was employed. It gives a complex harmonic E. M. F. in which the third and fifth harmonic are quite strong. At the normal speed the frequency of the fundamental was 600 p.p.s. By transformation as indicated in Fig. 10 the upper

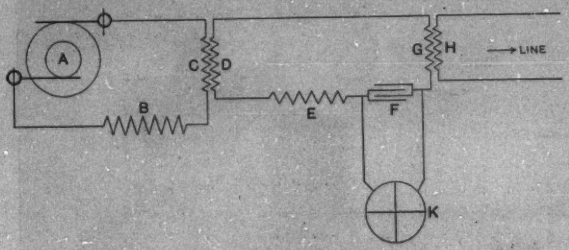


FIG. 10.

harmonics were weeded out by tuning, In this diagram A is the alternator. The secondary circuit D, E, F, G, contains a condenser F, and an auxiliary coil E. By adjusting the capacity of condenser F and the inductance of coil E, the impedance of this secondary circuit for the fundamental frequency can be reduced to a minimum, in which case the fundamental current in this circuit is by far predominant. Under these conditions the E. M. F. impressed upon the tertiary coil H, which connects to the line, is very nearly simple harmonic. This was verified by plotting experimentally the E. M. F. curve in the tertiary circuit. The point of resonance was detected by connecting the terminals of condenser F to a multicellular voltmeter K and watching for the maximum rise of the potential. While the observations which will be described presently were made the reading of this voltmeter was continually watched. A change in it indicated a change in speed

and impressed E. M. F. and the observations had to be suspended until the voltmeter returned to the same reading. Considering the variability of the electrical pressure in a University plant where every one of the many departments draws its current from the same source, it will be readily seen that these observations were somewhat trying.

2. To determine the mean value of the current a Townsend contact-maker (G, Fig. 8) was used.[1] Its circumference was divided into thirty equal parts, since the alternator had thirty poles. Every other section of the circumference was connected to a ring g. This ring and the circumference were connected to the rest of the circuit, $g\ F\ H\ h$, by two brushes $g\ h$. The contact-maker was mounted on the shaft of the alternator. The time interval, of half a period, during which the contact lasted, could be shifted by shifting the brush h along the circumference of the contact-disk. H is a Rowland-D'Arsonval galvanometer suitably shunted so as to keep the readings within a desirable range. The current readings were obtained as follows: A two-pole contact spring-jack (1, Fig. 8) was inserted across the terminals of an inductance coil, say a_1, as indicated. The coil was thus shunted by circuit $1\ E$. This circuit contained a small coil e and one or two ordinary 16-c.-p. incandescent lamps in series with it. The coil acted inductively upon the circuit containing the contact-maker and the galvanometer. A simple consideration will show that the galvanometer reading was proportional to the mean value of the current flowing through the inductance coil a_1 during the half-period corresponding to the position of the brush h. Suppose now that this brush is set in such a way as to give the maximum reading when the contact-spring-jack is across the inductance coil at the middle point of the cable. Then, as shown in equation (1), the reading at any other inductance coil will be proportional to

$$\left(e^{\beta\xi} + e^{-\beta\xi}\right) \cos \frac{2\pi}{\lambda} \xi$$

where ξ denotes the distance of the inductance coil under consideration from the middle point. This expression passes through zero whenever ξ is an odd multiple of the quarter wave-length, and the reading passes then from one side of the zero position to the other. The brush h was, therefore, set so as to give the maxi-

[1] See TRANSACTIONS, vol. xvii., No. 1, Jan., 1900.

mum reading at the middle point of the line. Then the spring-jack was run along the line and the sign of the readings noted at 1, 2, 3,.... etc. The number of reversals of readings corresponding to a given length of the line was counted.

The reversals occur whenever ξ is an odd multiple of $\lambda/4$. Suppose that five reversals occurred in the distance of 41 miles. It means that 9/4 wave-lengths cover the distance of 41 miles; hence the wave-length is

$$\lambda = 18.2 \text{ miles.}$$

This was found to be the case with a frequency of 600 P. P. S. The wave-length calculated from the formula

$$\lambda = \frac{2\pi}{p\sqrt{2LC}}$$

$$p = 2\pi \times 600.$$
$$L = .058.$$
$$C = .074 \times 10^{-6}$$

gives $$\lambda = 18.1 \text{ miles.}$$

The agreement between the calculated and the observed wave-length was extremely good for this frequency, considering the inequality of the cable sections which necessitates taking for L and C average values. Following is a table of wave-lengths for other frequencies, giving both the observed and the calculated values.

TABLE OF WAVE LENGTHS.

FREQUENCY.	$\lambda_{obs.}$	$\lambda_{calc.}$
625	17	17.3
600	18.2	18.1
450	24	24.1
260	41.4	41.7
245	44	44.4
230	48	48.2

At the frequency of 600 P.P.S. the non-uniform cable has 18 coils per wave-length. At the lower frequencies the number of coils per wave-length is larger, and therefore the cable approximates then its corresponding uniform conductor even more closely

than at the frequency of 600 P.P.S. But the approximation at this higher frequency is already so close that no difference in the behavior of the cable at the various frequencies experimented with and given above could be detected experimentally. As far as the wave-length determination is concerned the agreement between theory and experiment is very satisfactory.

3. To determine the wave-length on the cable without the coils plugs (see *e*, Fig. 9) were put in 1, 2, 3.... The readings were taken as follows: The two-pole spring-jack was inserted across a

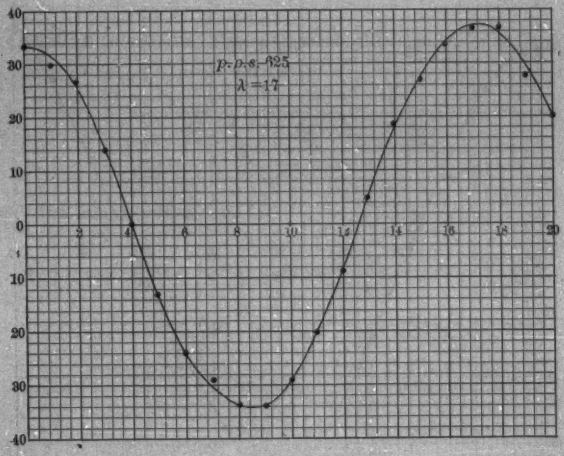


FIG. 11.

pair of plates, say 1. Then the short-circuiting plug (see *e*, Fig. 9) was taken out and the deflection in the galvanometer noted. The number of reversals was counted as before, and from these the wave-length was estimated. The value of the wave-length thus determined for a frequency of 600 P.P.S. was 126. Its calculated value was 125. Thus the insertion of the coils reduces the speed of propagation to one-seventh of its original value since it shortens the wave-length to that extent.

d. Determination of the Attenuation Constant — First

Method.—It was shown above that the equation of the curve of the mean value of current is

$$y = \frac{y_0}{2}\left(e^{\beta\xi} + e^{-\beta\xi}\right)\cos\frac{2\pi}{\lambda}\xi$$

where y is the mean value of the current at the distance ξ from the middle point of the cable and y_0 is the mean value at the

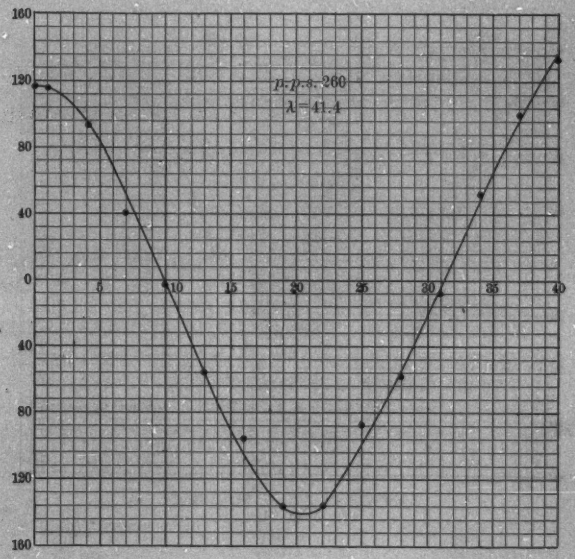


Fig. 12.

middle point when the brush h of the contact-maker is set to give a maximum deflection at this point. In Fig. 11 and Fig. 12 two such curves are given. In these curves the abscissæ represent the number of the coil at which the reading was taken, and the ordinate represents the galvanometer reading at the corresponding coil.

An attempt was first made to determine β from these curves.

Let $y_{\frac{\lambda}{2}}$ be the value of y at $\xi = \frac{\lambda}{2}$, then

$$y_{\frac{\lambda}{2}} = -\frac{y_0}{2}\left(e^{-\frac{\beta\lambda}{2}} + e^{-\frac{\beta\lambda}{2}}\right)$$

Since $\frac{\beta\lambda}{2}$ is small we can also write

$$y_{\frac{\lambda}{2}} = -\frac{y_0}{2}\left(2 + \frac{1}{4}\beta^2\lambda^2\right)$$

Therefore considering the numerical values, only, of y_0 and $y_{\frac{\lambda}{2}}$ we shall have

$$\beta = \frac{2}{\lambda}\sqrt{\frac{2(y_{\frac{\lambda}{2}} - y_0)}{y_0}}$$

Referring now to curve Fig. 11

$$\lambda = 17$$
$$y_0 = 33.5$$
$$y_{\frac{\lambda}{2}} = 34$$

$$\frac{2}{\lambda}\sqrt{\frac{2(y_{\frac{\lambda}{2}} - y_0)}{y_0}} = \frac{2}{17}\sqrt{\frac{1}{33.5}} = .02 = \beta$$

The value of β calculated from the formula

$$\beta = \frac{R}{\sqrt{2}\,10^3}\sqrt{\frac{C}{L}}$$

gives $\beta = .0141$

if we assume for L, C, and R the following average values: $L = .058$, $C = .074 \times 10^{-6}$, $R = 17.5$.

The question now arises: What degree of accuracy can be claimed for this experimental determination of β? The expression for β contains the difference of quantities which are

nearly equal. Now this difference is very near the limit of the errors of observation, the limit being quite considerable on account of several reasons. The chief reason is this. The condenser sections of the cable are far from uniform. This lack of uniformity is due, first to the varying thickness of the tinfoil which makes the resistance vary from section to section. Secondly, the sections are heated before compression is applied to them. This heating is of course not quite uniform; the amount of paraffin driven out by the applied pressure is, therefore, different in different sections. Hence the variation of capacity. The result is that the speed of propagation is different in different sections. The difference may amount to as much as 20 per cent. Hence the wave-length will change considerably from point to point of the cable. This will be observed in the curves of Fig. 11 and Fig. 12. Thus in Fig. 11 the first quarter wave-length is four miles long, the second quarter wave-length is 4.5 miles and the average is 4.25. The same variation will be observed in curve of Fig. 12. It is evident, therefore, that the method described above for taking readings at the various points of the cable will not give readings which will follow very strictly the mathematical law of the formula on page 22. Taking all these things into consideration, it is evident that this method just pointed out for determining experimentally the attenuation constant, is not strictly applicable to our cable.

Another method will be described now which determines the attenuation constant not from the one-half wave-lengths, only, near the middle of the line, but takes all the waves corresponding to a long length of the line into consideration. The wave-length determination given above agreed so much better with the theory for the very reason that the method employed gave the average wave-length and not the wave-length at any particular point of the cable.

Although the curves of Figs. 11, 12 and 13 do not enable us to calculate β accurately they are still of considerable importance. *First*, they show that the cable, though it has a large resistance, possesses, nevertheless, a small attenuation constant. This is particularly well shown in the curve of Fig. 13. This curve represents the mean square of current curve at the the various coils. The readings were obtained in this case by dispensing with the sliding contact arrangement and putting in place of E an electrodynamometer. This mean square curve represents a very near

approach to the stationary wave form, which shows that the attenuation is extremely small. *Secondly*, the smoothness of the curves 11, 12, and 13 shows that the non-uniform cable acts, even at the frequency of 625 P. P. S., just like a uniform cable. This, in fact, was the principal aim of this experimental research.

Second Method of Determining β.—The value of β was determined from

$$y = \frac{y_0}{2}\left(e^{\beta\xi} + e^{-\beta\xi}\right)\cos\frac{2\pi}{\lambda}\xi$$

as follows: Suppose that at a considerable distance from the

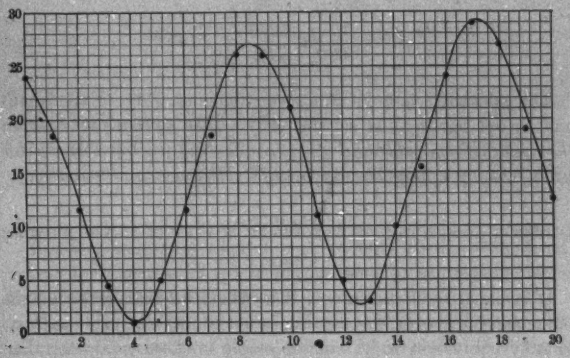


FIG. 13.

point $\xi = 0$ we find a point the distance of which is some multiple of $\lambda/2$. Then

$$y_1 = \frac{y_0}{2}\left(e^{\beta\xi_1} + e^{-\beta\xi_1}\right)$$

This was done experimentally by shifting the brush h (Fig. 8) until the maximum reading was obtained at the middle point of the line. It was 152 scale divisions—that is, $y_0 = 152$. Then the spring-jack was carried to the beginning of the cable and passed along from coil to coil until the galvanometer reading passed through a maximum. The distance ξ_1 of this point from $\xi = 0$ was 145. The reading actually obtained was 60, but this reading

was obtained from a shunt wire, and the reading as calculated from the shunt was 630. Now, when ξ_1 is so large that the omission of $e^{-\beta\xi_1}$ in comparison to $e^{+\beta\xi_1}$ involves an error of the same order of magntude as the errors of observation, we have the simplified formula

$$y_1 = \frac{y_0}{2} e^{\beta\xi_1}$$

Therefore,

$$\beta = \frac{1}{\xi_1} \log \frac{2\, y_1}{y_0}$$

In the case before us

$$\beta = \frac{1}{145} \log \frac{1260}{152} = .0145$$

The calculated value of β as given above is .0141. Experiment and theory agree quite satisfactorily. *The equivalence between the non-uniform cable and its corresponding uniform conductor up to 625 p.p.s. has therefore been established experimentally.*

Internal Reflection of Electrical Waves in a Non-uniform Cable.

It remains to be shown now that when the distribution of the inductance coils over the cable does not render the cable approximately equivalent to its corresponding uniform conductor, then there will be serious internal reflections which will diminish the efficiency of wave transmission.

For this purpose the inductance coils were connected in groups of ten; one of these groups was inserted into the cable every ten miles, so that the total inductance of the cable was now the same as before. A simple harmonic E. M. F. of 260 p. p. s. was impressed upon the cable. The wave-length of a wave of this frequency on the corresponding uniform conductor was found (see Fig. 12) to be 41.4 miles so that the inserted inductance coils were apart at a distance which is equal to about one-quarter of the wave-length on the corresponding uniform conductor. The degree of equivalence between this non-uniform cable and its corresponding uniform conductor is the same as the degree of equivalence between $\sin \pi/4$ and $\pi/4$; that is to say, the resemblance between the two is very small.

Theory of the Experiments on Internal Wave Reflections.—The current at any point between two consecutive groups of coils can be written as follows:

$$y = A\, e^{\beta\xi} \sin (pt - a\,\xi) + B\, e^{-\beta\xi} \sin (pt + a\,\xi)$$

that is, there are two waves, one a direct one, and the other a reflected one. This equation can also be written:

$$y = \sqrt{A^2\, e^{2\beta\xi} + B^2\, e^{-2\beta\xi} + 2\,A\,B\,\cos 2\,a\,\xi}\, \sin\,(pt - \varepsilon)$$

where

$$\tan \varepsilon = \frac{A\, e^{\beta\xi} - B\, e^{-\beta\xi}}{A\, e^{\beta\xi} + B\, e^{-\beta\xi}} \tan a\,\xi$$

Let $M\,(y)$ stand for the mean value of the current, then

$$M\,(y) = \frac{1}{\pi} \sqrt{A^2\, e^{2\beta\xi} + B^2\, e^{-2\beta\xi} + 2\,AB\cos 2a\xi}\, \cos\,(pt_1 - \varepsilon)$$

This is a maximum when t_1 is given such a value that $pt_1 - \varepsilon = o$. In that case $M\,(y)$ is proportional to the square root of the mean square of the current.

Experiment.—The curve of $M\,(y)$ was obtained in the same manner as the curves of Figures 11 and 12, except in this case the movable brush h, Fig 8, was shifted until the galvanometer showed the maximum deflection at every reading. These readings were plotted as ordinates of curve Fig. 14. Points $a\,b\,c\,d$ on the axis of abscissæ mark the positions of the groups of inductance coils. The effect of reflections is shown in a very marked manner by the sharp corners and strong depressions. The sharp corners coincide, of course, with the positions of the coils. The rapid rise of the minima points show strong attenuation. To compare the attenuation in this case with the attenuation which exists where the coils are distributed one per mile it is only necessary to obtain from curve 14 the curve of mean square of current and compare it to curve of Fig. 13. Draw then a straight line through two consecutive minima and compare the slope of this line in curve 13 to that of curve 14. It will be found that the

slope in this latter case is very much more rapid, and, therefore, the attenuation is very much more powerful; and it should be remembered that in curve of Fig. 14 we have the attenuation which is accompanying a frequency of 260 p.p.s. With a frequency of 625 p.p.s. (the frequency of Fig. 13) the attenuation would be even much more powerful. I intend to continue this particular part of the research in the very near future.

Telephony Over the Non-Uniform Cable.—These experimental results leave no doubt as to the correctness of the prediction which the mathematical theory of Part II makes with regard to telephony over non-uniform cables. Actual experimental tests with transmitters and receivers which were kindly put at my disposal by the New York Telephone Company, for which I feel very grateful to them, verified these predictions completely. Telephonic communication over the whole of the cable was carried on with perfect ease. Both the volume and the articulation of the transmitted sound were all that could be desired. If, however, the inductance coils were cut out and the cable used as a uniform cable, then the transmission was good up to 50 miles, fair up to 75 miles, impracticable at 100 miles, impossible at distances over 112 miles. Increased distance interferes with the transmission over the uniform cable, not only on account of the diminished volume of the sound transmitted, but also on account of the rapid loss of articulation. This manifests itself at first as an apparent lowering of the pitch of the voice just as the theory demands it. My assistant's voice is of an ordinary pitch, if anything a little above the pitch of the average man's voice. When it was transmitted over 75 miles of the uniform cable (that is, the coils being disconnected from it) it sounded like a strong baritone. Over a distance of 100 miles it became so drummy, (this is an expression borrowed from telephone engineers) that it was difficult to understand it, unless nothing but ordinary conversation was carried on and then very slowly, and the speaker had to put his mouth as close to the transmitter as possible and speak out as if he were addressing a big audience. At distances over 112 miles nothing but the lowest notes of the voice could be heard; the articulation was entirely gone. When the coils were all in, then no drumminess in the voice was noticed, and the conversation could be carried on as rapidly as one chose to do. I understand that in long distance work three galvanic cells of the same type of which I had two only were formerly employed by the

American Telegraph and Telephone Company when they employed the old system of transmission.

Effect of internal reflections upon telephonic transmission.— The effect of internal reflection is illustrated even more strikingly by telephonic transmission than it is by the exploration of the simple harmonic wave as represented in Fig. 14. The same distribution of coils was used for telephonic transmission as described above in connection with the internal reflection experiments. A distance of a hundred miles of the cable was employed.

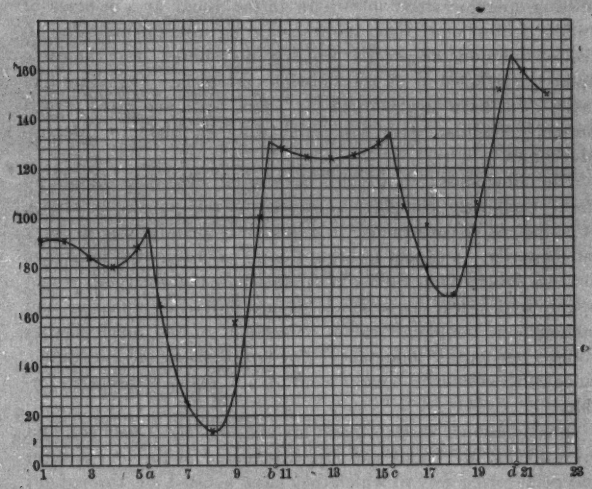


FIG. 14.

Telephonic transmission over this distance was absolutely impossible, although the lowest note in the voice was transmitted quite vigorously, and sounded like the muffled beating of a big drum. Another distribution of the coils was now tried. The coils were divided into groups of five, and these groups were put into the cable at points five miles apart. Transmission over a distance of a hundred miles became intelligible, though with much difficulty; but it was not as good as over the cable without the coils. The drumminess was even more excessive in the for-

mer case than in the latter. One had to put his mouth very close to the transmitter, speak very slowly, very loud, and about very ordinary things, otherwise the meaning of the transmitted intelligence would not be caught at all. But if the same distance of cable was employed with the inductance coils placed one mile apart as the mathematical theory suggests then the voice of the sender sounded just as if he stood right before the receiving operator. It was not necessary to stand near the transmitter at all, in order to be heard at the other end. There was no difficulty in making the receiving operator understand even if the transmitting operator stood anywhere in the room at a distance of twenty feet or more from the transmitting instrument. The effect of the inductance coils properly distributed over the cable upon the efficiency of transmission astonishes even one like myself who have now worked for years upon and made myself perfectly familiar with this fascinating problem of Electro-Mechanics.

The inductance coils which I have so far described contain no iron. Coils with iron cores to be used for this purpose will have, of course, to be constructed in a perfectly definite way to work satisfactorily. I expect to bring this matter before the INSTITUTE at some future time. Suffice it to state now that such coils would be required in the construction of non-uniform submarine and underground telephone cables on account of the much smaller volume per unit of inductance. For long distance air lines inductance coils without iron cores would be in many respects preferable.

In conclusion I wish to thank my assistant Mr. Cushman, and my students, Mr. Frank and Mr. Koscherak for the very faithful and persevering way in which they have assisted me throughout these experimental researches.

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PART II.[1]

Wave Propagation over Non-uniform Electrical Conductors.

Introduction.

The main object of the mathematical theory developed in this paper is the analysis of the propagation of electrical waves over a conductor represented in Fig. 3, Sec. III. This conductor consists of a loop of wire, $L_1 \ldots L_{k+1}$ of length $2l$, in which there are at equi-distant points, the so-called reactance points, a certain number of equal coils interposed dividing the loop into a number of equal parts, called the *interstices* of the loop. These coils may have condensers in series with them, or they may have secondary circuits with condensers. The propagation of electrical waves over a periodically loaded loop of this kind is compared to that over a uniform loop having the same total inductance, resistance, and capacity. This uniform loop is called "*the corresponding uniform conductor*" of the periodically loaded loop.

A similar problem in Mechanics is that of forced and of free vibrations of a periodically loaded heavy, flexible, inextensible string of finite length taking frictional resistance into account. To my knowledge, neither the electrical problem nor its corresponding mechanical problem have been investigated before.

It will be observed in the course of this paper that a study of the propagation of electrical waves over a periodically loaded conductor of this kind suggests forcibly an electromagnetic theory of emission and absorption of light by molecular complexes, which, on account of the physical conception underlying it, if for no other reason, possesses many attractive features. In this theory the ether and the material ions imbedded in it correspond to the uniform wire and the reactance points considered in this paper. These matters, however, are of a more or less speculative character, and have, therefore, no place here. But it should be noted that the physical problem discussed here was first suggested by speculations of this kind. It can be stated as follows: Under what conditions will the non-uniform conductor repre-

1. Reprint from the *Transactions* of the American Mathematical Society, Vol. I.

sented in Fig. 3 be approximately equivalent to its corresponding uniform conductor? Or, to be more precise: For what frequencies will an electrical wave have approximately the same wave length and the same damping or attenuation constant on one conductor as on the other?

The mathematical theory developed here gives a definite answer to this question. This answer can be stated in a few words, and for that purpose it is desirable to introduce here a convenient technical term, the so-called *angular distance* of the interstices—that is, of the intervals between consecutive reactance points. This interval is at any given frequency a definite fraction of the wave length corresponding to that frequency. Let this fraction be $\varphi/2\pi$; then φ is the angular distance of the interstices. The angular distance of a wave length is, of course, 2π. The general rule expressing the conditions of equivalence of a non-uniform conductor to its corresponding uniform conductor can now be expressed as follows: *For any given frequency a non-uniform conductor of the second type is equivalent to its corresponding uniform conductor as nearly as* $\sin \varphi/2$ *is to* $\varphi/2$ *itself.*

The higher the frequency the less resemblance will there be between a given non-uniform conductor and its corresponding uniform conductor. Conversely, if this resemblance is sufficiently close for a given frequency, it will be closer for all lower frequencies. When the half-wave length under consideration becomes smaller than the interstices, then the resemblance as far as that wave length and all shorter wave lengths are concerned ceases altogether.

A brief summary of this mathematical paper will be given now.

The main object of this research is the solution of the problem of Section III. This solution depends on the solution of the problem of Section II, and this again is moulded after the pattern of the solution of the problem of Section I.

In Section I the wave propagation over a uniform wire conductor represented in Fig. 1 is discussed. The effect of the transmitting apparatus A and of the receiving apparatus B is taken into account. Equation [5] is the most general solution of this case. The propagation of waves of both forced and of free periods is easily deduced from it. This particular form of the general solution is new, and it was selected because it is

best suited for comparing the propagation of waves over non-uniform conductors of Sections II and III to that over their corresponding uniform conductors.

In Section II the wave propagation over a non-uniform conductor of the first type represented in Fig. 2 is discussed. This conductor consists of a certain number of equal coils, $L_1 \ldots\ldots\ldots\ldots L_n$, which are connected in series. A certain number of equal condensers, one at each juncture between two consecutive coils, connect this conductor to ground. A transmitting apparatus A and a receiving apparatus B are present. The general solution of this problem, equation (6a), is moulded after the pattern of equation (5) of Section I. Both forced and free oscillations on a conductor of this kind are considered, and the conditions under which it becomes equivalent to its corresponding uniform conductor are worked out at considerable length. The problem of this section and its solution are both new.

The problem of Section III can, from a purely mathematical point of view, be stated as follows: Find the integral of the following partial differential equation:

$$L\frac{d^2y}{dt^2} + R\frac{dy}{dt} = \frac{1}{C}\frac{\partial^2 y}{\partial s^2}$$

and determine it in such a way as to satisfy $k+2$ boundary conditions. Determine also the conditions under which this integral will be equivalent to the integral represented in equation (5), Sec. I.

Equation (2) represents the most general solution, and the constants $x_1\ x_2 \ldots\ldots\ldots\ldots x_{k+2}$ have to be determined from the $k+2$ boundary conditions. The principal mathematical difficulty here reduces itself, then, to the proper mathematical formulation of these boundary conditions so as to obtain a system of equations which can be readily solved. Such a system is system (3) of this section. It is of the same form as system (6) of Section II, the solution of which was obtained in that section. Equations (4) and (5) are thus obtained. Equation (5) is the most general solution, and when this equation becomes very nearly the same as equation (5) of Section I, then the non-uniform conductor of this section becomes equivalent to its corresponding uniform conductor. This equivalence cannot be decided without a careful study of the wave lengths and the damping constants of waves of different periods. This study is

recorded in the remaining portion of this section. This section is also entirely new.

Sec. I and a part of Sec. II, together with a description of experimental investigations bearing upon the same, were published in Vol. XVI of the TRANSACTIONS of the AMERICAN INSTITUTE of ELECTRICAL ENGINEERS for 1899. Additional experimental investigations bearing upon the principal problem—that is, the problem of Sec. III, will be published in the near future.

SECTION I

WAVE PROPAGATION OVER A UNIFORM LINEAR CONDUCTOR.

A.—WAVES OF FORCED PERIOD.

The conductor is a loop of wire A, B, (Fig. 1.) At one point of the loop is a transmitting apparatus A, at the opposite point is a receiving apparatus B. The distance between A and B is l, equal to one-half the length of the whole loop. The distance of any element ds from A is s.

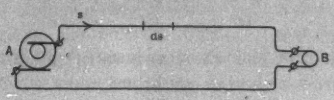


FIG. 1.

Let L, R, C, denote the inductance, resistance and capacity, respectively, per unit length. Let y be the current at any point s. Then

$$L\frac{d^2y}{dt^2} + R\frac{dy}{dt} = \frac{1}{C}\frac{\partial^2 y}{\partial s^2} \qquad (1)$$

is the well known equation of propagation.

To formulate the two boundary conditions, one at the receiving and the other at the transmitting apparatus, let L_0, R_0, C_0, and L_1, R_1, C_1, be the inductance, resistance, and capacity, respectively, of the transmitting and of the receiving apparatus. Each contains a condenser in series with its inductance and resistance. Let the E. M. F. impressed by A be of type Fe^{ipt}. The boundary equations can now be expressed as follows:

$$(2)\quad \begin{cases} -2\left(\dfrac{\partial y}{\partial s}\right)_{s=o} = \left[D_0 - h_0\, y\right]_{s=o} & \text{at transmitting apparatus.} \\ +2\left(\dfrac{\partial y}{\partial s}\right)_{s=l} = -\left(h_1\, y\right)_{s=l} & \text{at receiving apparatus.} \end{cases}$$

where

$$D_0 = ip\,C\,E\,e^{ipt}$$

$$h_0 = C(-p^2\lambda_0 + i\,p\,R_0),\ h_1 = C(-p^2\lambda_1 + i\,p\,R_1)$$

$$\lambda_0 = L_0 - \frac{1}{p^2 C_0},\quad \lambda_1 = L_1 - \frac{1}{p^2 C_1}$$

The physical character of the problem suggests the following solution:

$$y = K_1 \cos\mu\,\xi + K_2 \sin\mu\,\xi, \tag{3}$$

where $\xi = l - s$, and K_1 and K_2 are proportional to e^{ipt}.

Equation (1) is satisfied for all values of K_1 and K_2 if

$$C(-p^2 L + i\,p\,R) = -\mu^2 = -(\alpha + i\,\beta)^2$$

From this we get

$$\alpha = \sqrt{\tfrac{1}{2}\,p\,C\,[\sqrt{p^2 L^2 + R^2} + p\,L]}$$

$$\beta = \sqrt{\tfrac{1}{2}\,p\,C\,[\sqrt{p^2 L^2 + R^2} - p\,L]} \tag{4}$$

When $p\,L$ is large in comparison to R,

$$\alpha = p\,\sqrt{L\,C},\quad \beta = \frac{R}{2}\sqrt{\frac{C}{L}}. \tag{4a}$$

The boundary equations will be satisfied if

$$K_1 = \frac{2\,\mu\,D_0}{F},\qquad K_2 = \frac{h_1\,D_0}{F},$$

where $F = (h_0 h_1 - 4\,\mu^2)\sin\mu\,l + 2\,\mu\,(h_0 + h_1)\cos\mu\,l$

* Observe that in the case of a twin conductor where the mutual capacity alone is considered, we must use $2\,R$ and $2\,L$ in place of R and L.

Hence (3) can now be written

$$y = (2\,\mu \cos \mu\,\xi + h_1 \sin \mu\,\xi)\,\frac{D_0}{F} \tag{5}$$

When $h_0 = h_1 = 0$

$$y = -\frac{D_0 \cos \mu\,\xi}{2\,\mu \sin \mu\,l} \tag{5a}$$

Equation (5) is a complete solution for the propagation of waves of forced period. It represents simple harmonic damped waves. The most essential elements which enter into the description of such waves are the *wave length* λ and the *damping* or *attenuation factor*. These can easily be calculated. Since $\mu = a + i\,\beta$, we shall have $\lambda = 2\,\pi/a$; the attenuation factor is $e^{-\beta\xi}$

It is evident that β diminishes as L increases. A high inductance per unit length means small attenuation and a slow speed of propagation.

B.—Waves With a Natural Period.

Free oscillations are readily calculated for a few special cases. Equation (5) is a general solution for free oscillations also, provided, however, that μ has such a value as to make

$$F = 0, \text{ since } D_0 = 0;$$

that is, we must have

$$(h_0\,h_1 - 4\,\mu^2) \sin \mu\,l + 2\,\mu\,(h_0 + h_1) \cos \mu\,l = 0, \tag{6}$$

but of course, in this case,

$h_0 = k\,C(k\,\lambda_0 + R_0)$, $h_1 = k\,C(k\,\lambda_1 + R)$, $-\mu^2 = k\,C(k\,L + R)$.

Equation (6) is a transcendental equation and can be readily solved in a few simple cases.

Case 1.—The transmitting and receiving apparatus are not present. In this case $h_0 = h_1 = 0$. Equation (6) reduces to $\sin \mu\,l = 0$, therefore $\mu = s\,\pi/l$, where s can have any integer value from 1 to ∞. The periods of free oscillations are calculated from the equation

$$-\mu^2 = k^2\,L\,C + k\,R\,C = -\frac{s^2\,\pi^2}{l^2}.$$

Therefore

$$k = -\frac{R}{2L} \pm \sqrt{-1}\sqrt{\frac{1}{LC}\frac{s^2\pi^2}{l^2} - \frac{R^2}{4L^2}}$$

$$= -\frac{R}{2L} \pm i\,k_s.$$

There are, therefore, an infinite number of periods which are harmonically related to each other unless the damping constant $R/2\,L$ is not negligibly small in comparison with $\pi^2/l^2\,L\,C$. The most general solution of this case can be written

$$y = e^{-\frac{Rt}{2L}} \sum_{s=1}^{\infty} A_s \cos\left(\frac{s\,\pi}{l}\,\xi\right) \cos\left(k_s\,t - \varepsilon_s\right) \qquad (7)$$

The wave lengths are

$$\frac{2\,l}{1},\ \frac{2\,l}{2},\ \frac{2\,l}{3},\ \ldots\ \frac{2\,l}{s},\ \ldots\ldots$$

Case 2.—Transmitting apparatus is not present, and in place of the receiving apparatus there is a break in the wire.

In this case $h_0 = 0$, $h_1 = \infty$. Equation (6) reduces to

$$\cos \mu\,l = 0.$$

Therefore

$$\mu = \frac{2\,s+1}{l}\,\frac{\pi}{2},$$

$$k = -\frac{R}{2L} \pm i\sqrt{\frac{1}{LC}\left(\frac{2\,s+1}{l}\,\frac{\pi}{2}\right)^2 - \frac{R^2}{4L^2}}$$

$$= -\frac{R}{2L} \pm i\,k_{2s+1},$$

$$y = e^{-\frac{Rt}{2L}} \sum_{0}^{\infty} A_{2s+1} \sin\left(\frac{2\,s+1}{l}\,\frac{\pi}{2}\,\xi\right) \cos\left(k_{2s+1} - \varepsilon_{2s+1}\right) \qquad (8)$$

The wave lengths are $\frac{4\,l}{1},\ \frac{4\,l}{3},\ \frac{4\,l}{5},\ \ldots\ldots$

The damping factor is in both cases the same for all frequencies, hence the color of the complex harmonic vibration remains unchanged during the whole epoch of its existence.

SECTION II.

ELECTRICAL OSCILLATIONS ON A NON-UNIFORM CONDUCTOR OF THE FIRST TYPE.

The conductor consists of $2n$ equal coils $L_1, L_2 \ldots L_n$ (Fig. 2) connected in series, so as to form a closed loop. At one point A of this loop is an alternator, at the opposite point is a receiving apparatus, B. At equal distances $2(n-1)$ equal condensers, $C_1 \ldots\ldots C_{n-1}$, connect the conductor to ground.

The whole loop is thus divided into $2n-2$ component circuits 1, $2 \ldots\ldots 2n-2$. It is evident that in the limit when n becomes infinitely large, this conductor becomes an ordinary telegraph or telephone line with uniformly distributed resistance, capacity, and inductance. The question arises now, under what conditions will a conductor of this kind become

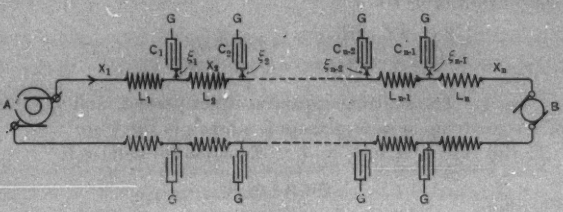


FIG. 2.

equivalent with sufficient approximation to a uniform line, even when n is not infinitely large? This problem does not seem to have been solved before.

In its main features it is similar to that which Lagrange solved in his "Mecanique Analytique, sec. partie, Sec VI," the problem, namely, of the free vibrations of a weightless string, fixed at its two ends and loaded at equi-distant points by equal weights. But it is much more general, because frictional resistances are taken into consideration, and also because forced as well as free oscillations are considered.

Let L_0, R_0, C_0, and L_1, R_1 and C_1, be the inductance, ohmic resistance, and capacity of A and B, respectively. Let L, R, C, be the corresponding quantities of the coils and condensers in the several component circuits.

The real part of $E e^{ipt}$ is the E.M.F. impressed by alternator A Let $x_1 \ldots\ldots x_n$ be the currents in the component circuits.

$P_1 \ldots\ldots P_{n-1}$ are the differences of potential in the line condensers, $C_1, C_2 \ldots\ldots C_{n-1}$; P_0 and P^1 are the differences of potential in the condensers in A and B, respectively; $\xi_1 \ldots\ldots \xi_{n-1}$ are the condenser currents. We shall have

$$\left.\begin{aligned} \xi_1 &= C\frac{d\,P_1}{dt}, \qquad \xi_2 = C\frac{d\,P_2}{dt}, \ldots\ldots \text{etc.} \\ \xi_1 &= x_1 - x_2, \qquad \xi_2 = x_2 - x_3, \ldots\ldots \text{etc.} \end{aligned}\right\} \quad (1)$$

A.—Forced Oscillations.

Stating the law of equality of action and reaction for each component circuit we obtain the following n differential equations:

$$\left.\begin{aligned} (L_0 + 2L)\frac{d\,x_1}{dt} + (R_0 + 2R)\,x_1 + 2P_1 + P_0 &= E\,e^{ipt}, \\ L\frac{d\,x_2}{dt} + R\,x_2 + P_2 - P_1 &= 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ L\frac{d\,x_{n-1}}{dt} + R\,x_{n-1} + P_{n-1} - P_{n-2} &= 0, \\ (L_1 + 2L)\frac{d\,x_n}{dt} + (R_1 + 2R)\,x_n - 2P_{n-1} + P^1 &= 0. \end{aligned}\right\} \quad (2)$$

When the steady state has been reached, the currents will be just like the impressed E. M. F., simple harmonics of the time t, that is,

$$x_1 = A_1\,e^{ipt}, \quad x_2 = A_2\,e^{ipt}, \quad (3)$$

where $A_1, A_2 \ldots\ldots$ are complex quantities.

Differentiating each member of (2) and substituting from (3) and (1), we obtain

$$\left.\begin{aligned} h\,x_1 + \xi_1 \quad - 0 \quad &= D, \\ h\,x_2 + \xi_2 \quad - \xi_1 \quad &= 0, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\ h\,x_{n-1} + \xi_{n-1} - \xi_{n-2} &= 0, \\ h\,x_n + 0 \quad - \xi_{n-1} &= -h_1\,x_0, \end{aligned}\right\} \quad (4)$$

where $$h = C(-p^2 L + i p R),$$

$$D = \tfrac{1}{2} i p C E e^{ipt} - \tfrac{1}{2} C(-p^2 \lambda_0 + i p R_0) = D_0 - h_0 x_1,$$

$$h_1 = \tfrac{1}{2} C[-p^2 \lambda_1 + i p R_1],$$

$$\lambda_0 = L_0 - \frac{1}{p^2 C_0}, \qquad \lambda_1 = L_1 - \frac{1}{p^2 C_1}.$$

Another form is obtained by substituting for $\xi_1, \xi_2 \ldots\ldots$ as follows:

$$\left.\begin{aligned}
&(h+1)\,x_1 + 0 - x_2 = D = D_0 - h_0 x_1 \\
&(h+2)\,x_2 - x_1 - x_3 = 0 \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&(h+2)\,x_{n-1} - x_{n-2} - x_n = 0 \\
&(h+1)\,x_n - x_{n-1} - 0 = -h_1 x_n.
\end{aligned}\right\} \quad (6)$$

Equation (3) of Sec. 1 suggests the following solution:

$$x_m = K_1 \cos 2(n-m)\theta + K_2 \sin 2(n-m)\theta \quad [6a]$$

If $h + 2 = 2\cos 2\theta$ then all the equations except the first and the last will be satisfied for all values of K_1 and K_2. These two equations, which correspond to the boundary equations of Sec. 1., will be also satisfied if we assign to K_1 and K_2 suitable values, as follows:

$$K_2 = \frac{(h_1 - 1) + \cos 2\theta}{\sin 2\theta} K_1$$

$$K_1 = \frac{D_0 \sin 2\theta}{h_0 h_1 \sin 2(n-1)\theta - 4\sin^2\theta \sin 2n\theta + 2(h_0 + h_1)\sin\theta\cos(2n-1)\theta}$$

we can now write, (7)

$$x_m = \frac{[2\sin\theta\cos(2n-2m+1)\theta + h_1 \sin 2(n-m)\theta]\, D_0}{h_0 h_1 \sin 2(n-1)\theta - 4\sin^2\theta \sin 2n\theta + 2(h_0+h_1)\sin\theta\cos(2n-1)\theta}$$

θ is a complex angle, hence the forced oscillations of type e^{ipt} on a non-uniform conductor of this kind are simple harmonic damped oscillations. The similarity between this conductor and a uniform wire will be discussed presently. When $h_0 = h_1 = o$ we obtain

$$x_m = \frac{-D_0 \cos(2n - 2m + 1)\theta}{2\sin\theta \sin 2n\theta} \quad (7a)$$

B.—Free Oscillations.

Equation (7) holds true for free as well as for forced oscillations. But since in the case of free oscillations $D_0 = o$ it follows that the denominator of (7) must vanish to prevent the vanishing of all currents. We shall have, therefore,

$$h_0 h_1 \sin (2n - 2)\theta - 4 \sin^2 \theta \sin 2n\theta + \\ + 2 (h_0 + h_1) \sin \theta \cos (2n - 1)\theta = o \quad (8)$$

From this equation θ and the corresponding periods and damping constants have to be determined. A solution can be readily obtained for a small number of cases. The two most important will be considered here.

First.—The transmitting and the receiving apparatus are not present. In this case $h_0 = h_1 = o$, and

$$x_m = B \cos (2n - 2m + 1)\theta \quad (9)$$

It is found from (8) that (9) is actually the solution of the differential equations (6) for $h_0 = h_1 = D_0 = 0$, provided that $\theta = \frac{s\pi}{2n}$, where s is any integer from 1 to $2n$.

Hence the most general solution will be

$$x_m = \sum_{s=1}^{2n} B_s \cos (2n - 2m + 1) \frac{s\pi}{n} \quad (10)$$

But it should be observed now that x_m is a periodic function of the time, that is

$$x_m = \sum_{s=1}^{\infty} K_s e^{p_s t}$$

Hence in (10) each amplitude B contains the time factor $e^{p_s t}$.

The constant p_s which measures the period and the damping constant of the free oscillation is determined from the relation:

$$h = -4 \sin^2 \theta.$$

In the case of free oscillations

$$h = p_s^2 L C + p_s R C, \theta = \frac{s\pi}{2n}.$$

Hence

$$p_s^2 L C + p_s R C = -4 \sin^2 \frac{s}{2n} \frac{\pi}{}. \qquad (11)$$

Before solving this equation, it is desirable to make the following substitution:

Let L', C', R' be the total inductance, capacity, and resistance, respectively, of one-half of the conductor, then

$$L = \frac{L'}{n}, \quad C = \frac{C'}{n}, \quad R = \frac{R'}{n}.$$

Let l denote the half length of a uniform wire having σ, r, c, for inductance, resistance, and capacity per unit length, and let it have the same total inductance, resistance, and capacity as the non-uniform conductor. Since

$$l\sigma = L', \; lr = R', \; lc = C',$$

we shall have

$$L = \frac{l\sigma}{n}, \; C = \frac{lc}{n}, \; R = \frac{lr}{n}.$$

This uniform wire having the same total inductance, resistance, and capacity as the non-uniform conductor, will be called the "*corresponding uniform conductor.*"

From (11) we obtain

$$\frac{l^2}{n^2} (p_s^2 \sigma c + p_s cr) = -4 \sin^2 \frac{s}{2n} \frac{\pi}{},$$

therefore

$$p_s = -\frac{r}{2\sigma} \pm \sqrt{-1} \sqrt{\frac{1}{\sigma c} \frac{4n^2}{l^2} \sin^2 \frac{s}{2n} \frac{\pi}{} - \frac{r^2}{4\sigma^2}},$$

$$p_s = \frac{r}{2\sigma} \pm i k_s$$

Equation (10) becomes now:

$$x_m = e^{\frac{-rt}{2\sigma}} \sum_{1}^{2n} {}_s A_s \cos (2n - 2m + 1) \frac{s}{2n} \frac{\pi}{} \cos (k_s t - \varepsilon_s). \qquad (12)$$

It is clear that the oscillations on the non-uniform conductor of type (1) have the same damping constant as the oscillations on the corresponding uniform conductor. It will be shown presently that under certain conditions they will also have the longer periods up to a certain limit very nearly the same as the corresponding uniform conductor. *For these periods, then, the non-uniform conductor will be equivalent to its corresponding uniform conductor.*

Second Case.—The transmitting apparatus is not present, and in place of the receiving apparatus there is a break in the line at B.

In this case $h_0 = 0$, $h_1 = \infty$. Equation (9) gives

$$x_m = B \sin (2n - m + 2)\theta,$$

provided that

$$\cos (2n - 1)\theta = o,$$

or

$$\theta = \frac{2s+1}{2n-1}\frac{\pi}{2}.$$

We shall have, therefore,

$$p_{2s+1} = \frac{-r}{2\sigma} \pm i\sqrt{\frac{1}{\sigma c}\frac{4n^2}{l^2}\sin^2\frac{2s+1}{2n-1}\frac{\pi}{2} - \frac{r^2}{4\sigma^2}}$$

$$= -\frac{r}{2\sigma} \pm i\,k_{2s+1}$$

Hence

$$x_m = e^{-\frac{r}{2\sigma}t}\sum_{0}^{2n} A_{2s+1}\sin(2n - 2m + 2)\frac{2s+1}{2n-1}\frac{\pi}{2}$$

$$\cos(k_{2s+1}\,t - \varepsilon_{2s+1}). \quad (13)$$

The same remark regarding the damping constant and the free periods applies here as in the preceding case.

The Wave Lengths of Free Oscillations.

The angles $s\pi/2n$ and $(2s+1)\pi/2(2n-1)$ have an interesting physical meaning which will be brought out by consid-

ering the wave-lengths of free oscillations. Consider one of the component harmonics of x_m in Case I, say

$$\xi_{m,s} = A_s \cos (2n - 2m + 1)\, s\pi/2n \cos (k_s t - \varepsilon_s).$$

Compare it to the corresponding component of x_{m_1}, that is to

$$\xi_{m_1,s} = A_s \cos (2n - 2m_1 + 1)\, s\pi/2n \cos (k_s t - \varepsilon_s).$$

If they are a wave length apart, $\xi_{m,s} = \xi_{m_1,s}$ and $m_1 - m =$ number of coils covered by one wave length. But in this case

$$(2n - 2m + 1)\, s\pi/2n = (2n - 2m_1 + 1)\, s\pi/2n + 2\pi,$$

therefore $$m_1 - m = 2n/s = \nu_s.$$

$\nu_s = 2n/s$ is, therefore, the number of coils covered by a wave length which corresponds to the harmonic s. It can be shown that in the second case

$$\frac{2(2n-1)}{2s+1} = \nu_s$$

For $\frac{s\pi}{2n}$ and $\frac{2s+1}{2n-1}\frac{\pi}{2}$ we can, therefore, write $\frac{\pi}{\nu_s} = \frac{1}{2}\frac{2\pi}{\nu_s}$

The physical meaning of $2\pi/\nu_s$ can now be readily fixed. A coil represents a definite fraction of a wave length, and this fraction will have a different value for different harmonics. The higher the harmonic the shorter will be the wave length, and therefore the larger will be the value of this fraction. It is convenient, however, to measure this fraction in terms of an angle instead of in terms of a wave length. If we arbitrarily assume that an angular distance 2π corresponds to a wave length, then an angular distance $2\pi/n$ will correspond to the nth part of a wave length. With this understanding

$$\frac{s\pi}{2n} \text{ and } \frac{2s+1}{2n-1}\frac{\pi}{2}, \text{ that is, } \frac{1}{2}\frac{2\pi}{\nu_s}$$

represents one-half of the angular distance covered by a coil.

A Non-Uniform Conductor of the First Type Compared to its Corresponding Uniform Wire.

a.—Similarity with Respect to Free Oscillations.

Comparing the expressions for the free periods of oscillation

which were obtained in Sections I and II we see that as long as π/ν_s can be written for $\sin \pi/\nu_s$ so long will the periods of free oscillations of the non-uniform conductor be nearly the same as those of its corresponding uniform conductor. We have, therefore, the simple rule: *A non-uniform conductor of the first type represents its corresponding uniform conductor as nearly as one-half of the angular distance covered by one of its coils represents the sine of that distance.* The non-uniform conductor employed by me[1] in my experiments had 400 coils. In this case

$$\frac{\pi}{\nu_s} = \frac{s\,\pi}{400}$$

For $s = 25$ we have $\pi\ \nu_s = \pi/16$. Now, $\sin \pi/16$ differs from $\pi/16$ by $\frac{2}{3}$ of one per cent. of the value of $\pi/16$. Hence, the period of the 25th harmonic of my non-uniform conductor differs from that of the same harmonic on the corresponding uniform wire by less than $\frac{1}{2}$ of one per cent. For lower harmonics the difference is smaller. The 25th harmonic had approximately 3500 P.P.S. Hence, up to the 25th harmonic—that is, up to 3500 P.P.S.—the non-uniform conductor employed by me had nearly the same free periods as its corresponding uniform conductor.

b.—SIMILARITY WITH RESPECT TO FORCED OSCILLATIONS.

The wave length and the attenuation constant corresponding to a given frequency speed p_s can be studied by studying the angle θ from the following equation:

$$-4 \sin^2\theta_s = -p_s^2\, L\, C + i\, p_s\, R\, C = \frac{l^2}{n^2}[-p_s^2\, \sigma\, C + i\, p_s\, r\, C]$$

$$= -\frac{l^2}{n^2}\,\mu_s^2 = -\frac{l^2}{n^2}\,(\alpha_s + i\,\beta_s)^2 \text{ or } \sin\theta_s = \tfrac{1}{2}\frac{l}{n}\,(\alpha_s + i\,\beta_s)$$

where σ, c, r are the inductance, capacity and resistance, respectively, per unit length of the corresponding uniform conductor; μ_s, α_s and β_s have the same meaning as in Sec. I.

Referring now to equation (4) Sec. I, it will be found that

$$\alpha_s \geq \beta_s.$$

1. See paper cited in the introduction.

If, therefore, $\frac{1}{2}\alpha_s/2n$ is sufficiently small then we can put

$$\theta_s = \tfrac{1}{2}\frac{l}{n}(\alpha_s + i\beta_s) = \tfrac{1}{2}\frac{l}{n}\mu_s.$$

When this substitution is made in (7) and (7ª) these equations will transform into (5) and (5ª) of Sec. I, which shows that under these conditions the non-uniform conductor of the first type becomes approximately equivalent to its corresponding uniform conductor, the degree of approximation being the same as the degree of equality between sin θ_s and θ_s.

The physical meaning of this can be readily made clear. If by λ_s we denote the wave length on the corresponding uniform conductor corresponding to the frequency speed p_s, then according to Sec. I,

$$\alpha_s = \frac{2\pi}{\lambda_s}, \quad \frac{l}{n}\alpha_s = \frac{l}{n}\frac{2\pi}{\lambda_s}.$$

Let n_s be the number of coils on the non-uniform conductor which cover a wave length of frequency speed p_s, and let φ_s be the angular distance covered by a coil, then

$$\varphi_s = \frac{2\pi}{n_s}.$$

Again $$n : n_s :: l : \lambda_s,$$

or $$n : \frac{2\pi}{\varphi_s} :: l : \lambda_s.$$

Hence $$\frac{l}{n}\frac{2\pi}{\lambda_s} = \frac{l}{n}\alpha_s = \varphi_s$$

The substitution mentioned above will therefore be permissible when $\frac{1}{2}\varphi_s$ (that is, one-half of the angular distance covered by a coil) is approximately equal to sin $\varphi_s/2$.

We have here the same rule as in the case of free oscillations. It will be shown in the next section that this rule is also applicable to non-uniform conductors of the second type for waves of both forced and of free periods.

SECTION III.

WAVE PROPAGATION OVER A NON-UNIFORM CONDUCTOR OF THE SECOND TYPE.

A. Waves of Forced Period.

Such a conductor is represented in diagram of Fig. 3. A long, uniform wire forms a loop $L_1 \ldots L_{k+2}$. At equal intervals are inserted in series $2k+2$ equal coils $L_1\ L_2 \ldots$. To make the discussion more general, let each coil have a condenser of capacity C_0 in series with it.

Let L, R, C represent the inductance, resistance, and capacity, respectively, per unit length of the uniform wire. Let an E. M. F. of type Ee^{ipt} be impressed at L. Employing the usual notation we shall have the following equation of propagation at every point of the uniform wire:

$$L\frac{d^2y}{dt^2} + R\frac{dy}{dt} = \frac{1}{C}\frac{\partial^2 y}{\partial x^2} \qquad (1)$$

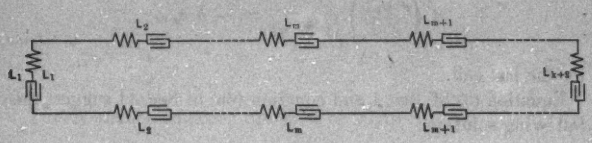


Fig. 3.

The same as in Sec. I, but here the integral has to satisfy 2 k + 2 boundary conditions, one at each coil. On account of the symmetry of distribution of the coils on the two sides of the loop this number is reduced to k + 2.

To formulate these boundary conditions introduce the following notation:

Let $2l$ = length of loop; $l/(k+1)$ will be the distance between two consecutive coils. Let L_0, R_0, C_0 be the inductance, resistance and capacity, respectively, of each coil. Let y_m be the current at any point of the interval between coils L_m and L_{m+1}. Denote the distance of this point from coil L_m by ξ.

The currents in the coils will be denoted by $x_1\ x_2\ x_{k+1}\ x_{k+2}$. It is evident that

$$(y_m)_{\xi=0} = (y_{m-1})_{\xi=l/k+1} = x_m.$$

The boundary conditions for all coils except L_1 and L_{k+2} can now be stated.

$$\left(\frac{\partial y_{m-1}}{\partial \xi}\right)_{\xi=l/k+1} - \left(\frac{\partial y_m}{\partial \xi}\right)_{\xi=0} =$$

$$= - C(-p^2 \lambda_0 + i p R_0) x_m = - h x_m,$$

where $$\lambda_0 = L_0 - \frac{1}{p^2 C_0}.$$

There are k equations of this form. The boundary equations for the first and the last coil are of the following form:

$$-2\left(\frac{\partial y_1}{\partial \xi}\right)_{\xi=0} = i p C E e^{ipt} - h x_1 = D - h x_1$$

for the first coil, and

$$2\left(\frac{\partial y_{k+1}}{\partial \xi}\right)_{\xi=l/k+1} = - h x_{k+2}$$

for the last coil.

Equation (3) of Sec. I and equation (6a) of Sec. II suggest the following solution:

$$y_m = K_1 \cos \mu \xi + K_2 \sin \mu \xi, \qquad (1a$$

where μ has the same value as in Sec. I.
When $\xi = 0$, y_m becomes x_m, and when $\xi = l/k + 1$, y_m becomes x_{m+1}.

Hence

$$x_m = K_1, \; x_{m+1} = K_1 \cos \frac{\mu l}{k+1} + K_2 \sin \frac{\mu l}{k+1}.$$

Therefore

$$y_m = \frac{x_m \sin \mu \left(\frac{l}{k+1} - \xi\right) + x_{m+1} \sin \mu \xi}{\sin \frac{\mu l}{k+1}} \qquad (2)$$

We have now to determine the $k + 2$ constants $x_1 x_2 \ldots x_{k+2}$

from the $k+2$ boundary equations. Begin with the boundary equation at the coil L_1:

$$\frac{\partial y_m}{\partial \xi} = \frac{\mu\left[-x_m \cos \mu\left(\frac{l}{k+1} - \xi\right) + x_{m+1} \cos \mu \xi\right]}{\sin \frac{\mu l}{k+1}}$$

Therefore

$$-2\left(\frac{\partial y_1}{\partial \xi}\right)_{\xi=0} = 2\left(x_1 \cos \frac{\mu l}{k+1} - x_2\right) \frac{\mu}{\sin \frac{\mu l}{k+1}} = D - h x_1$$

$$\text{Let } \sigma = h \frac{\sin \frac{\mu l}{k+1}}{\mu} - 4 \sin^2 \frac{1}{2} \frac{\mu l}{k+1},$$

$$\rho = \frac{\sin \frac{\mu l}{k+1}}{\mu} D;$$

This boundary equation can then be written:

$$\left.\begin{aligned}
&(\sigma+1)\, x_1 - 0 - x_2 = \frac{\rho}{2} + \frac{\sigma}{2} x_1 \\
\text{Similarly}\quad &(\sigma+2)\, x_2 - x_1 - x_3 = 0 \\
&(\sigma+2)\, x_3 - x_2 - x_4 = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
&(\sigma+2)\, x_m - x_{m-1} - x_{m+1} = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
&(\sigma+2)\, x_{k+1} - x_k - x_{k+2} = 0 \\
&(\sigma+1)\, x_{k+2} - x_{k+1} - 0 = \frac{\sigma}{2} x_{k+2}
\end{aligned}\right\} (3)$$

This system is of the same form as system (6) of Sec. II.

Equation (7) of that section enables us to write down the solution of (3).

Let $$\sigma + 2 = 2 \cos 2\,\phi$$

then

$$x_m = -\frac{D \sin \frac{\mu l}{k+1} \cos 2\,(k - m + 2)\,\phi}{2\,\mu \sin 2\,\phi \sin 2\,(k+1)\,\phi} \tag{4}$$

$$y_m = -\frac{[\cos 2(k-m+2)\phi \sin \mu\left(\frac{l}{k+1} - \xi\right) + \cos 2(k-m+1)\phi \sin \mu\xi]D}{2\,\mu \sin 2\,\phi \sin 2\,(k+1)\,\phi} \tag{5}$$

These equations give the complete mathematical representation of waves of forced period on a conductor of this kind. φ and μ are, of course, complex angles; hence, in order to calculate from these equations in any particular case the principal quantities with which we are concerned in wave propagation, namely, the wave length and the attenuation constant, it would be necessary to study separately the real and the imaginary parts of these equations. This study in its broadest aspect presents considerable mathematical complexity. It is, however, somewhat remote from the principal aim of this paper, which is to ascertain the conditions under which a non-uniform conductor of the second type is approximately equivalent to its corresponding uniform conductor; that is to say, for what frequencies will the two have approximately the same wave lengths and the same attenuation constants?

Equivalence of a Non-Uniform Conductor of the Second Type to its Corresponding Uniform Conductor.

An answer to this question is obtained by studying the following equation:

$$\frac{h_s \sin \frac{\mu_s l}{k+1}}{\mu_s} + 2 \cos \frac{\mu_s l}{k+1} = 2 \cos 2\,\phi_s,$$

or

$$\frac{h_s \sin \frac{\mu_s l}{k+1}}{\mu_s} - 4 \sin^2 \tfrac{1}{2} \frac{\mu_s l}{k+1} = -4 \sin^2 \phi_s$$

The subscript s denotes that the particular frequency speed p_s is considered.

$$\mu_s = \alpha_s + i\,\beta_s, \qquad \alpha_s \geq \beta_s.$$

If λ_s is the wave length corresponding to p_s on the uniform wire before the introduction of the reactance points, then

$$\frac{l}{k+1} a_s = \frac{l}{k+1} \frac{2\pi}{\lambda_s}.$$

Let θ_s be the angular distance between two consecutive coils for wave length λ_s, then

$$2\pi : \lambda_s :: \theta_s : \frac{l}{k+1}$$

$$\therefore \quad \theta_s = \frac{l}{k+1} \frac{2\pi}{\lambda_s} = \frac{l}{k+1} a_s$$

Hence, if θ_s is so small that $\sin \theta_s = \theta_s$ very nearly, then

$$\frac{l}{k+1} h_s - \left(\frac{l}{k+1}\right)^2 \mu_s^2 = -4 \sin^2 \psi_s$$

or

$$\left(\frac{l}{k+1}\right)^2 C[-p_s^2 (\rho + L) + i p (r + R)] = -4 \sin^2 \psi_s$$

where ρ and r are the inductance and resistance, respectively, per unit length of the inserted coils, so that $\rho + L$ and $r + R$ are the inductance and resistance, respectively, of the corresponding uniform conductor. This last equation can also be written:

$$\tfrac{1}{2} \frac{l}{k+1} (a_s' + i \beta_s') = \sin \psi_s$$

In this case also $a_s' \geq \beta_s'$. Let λ_s' be the wave length for frequency speed p_s on the corresponding uniform conductor, then

$$a_s' = \frac{2\pi}{\lambda_s'}.$$

Put $\varphi_s' = l\, a_s'/(k+1)$ then φ_s' is the angular distance between two consecutive coils, the angular distance of λ_s' being 2π.

If, therefore, $\frac{1}{2} \varphi_s'$ is so small that $\sin \frac{1}{2} \varphi_s' = \frac{1}{2} \varphi_s'$ very nearly, then

$$\psi_s = \tfrac{1}{2} \frac{l}{k+1} (a_s' + i \beta_s') = \tfrac{1}{2} \frac{l}{k+1} \mu_s',$$

very nearly.

Making this substitution in (4) we obtain equation (5*a*), Sec. I. The equation of wave propagation for a non-uniform conductor of the second type is under these conditions the same as that for its corresponding uniform conductor. The two will have approximately the same wave length and the same attenuation constant for the frequency speed p_s. *The degree of approximation is the same as the degree of approximation in the equality between*

$$\sin \varphi_s/2 \text{ and } \varphi_s/2.$$

A numerical example will illustrate this point more clearly.

Consider a uniform telegraph wire having the following constants per mile:

$$L = .004 \text{ henrys}$$
$$R = 7 \text{ ohms}$$
$$C = .01 \times 10^{-6} \text{ farads}$$

Introduce at each mile a coil having

$$L_1 = .036 \text{ henrys}$$
$$R_1 = 1 \text{ ohm}$$

The non-uniform conductor thus obtained will have for its corresponding uniform conductor a uniform wire having per mile

$$L + L_1 = .04 \text{ henrys}$$
$$R + R_1 = 8 \text{ ohms}$$
$$C = .01 \times 10^{-6} \text{ farads}$$

For a frequency of 1500 P.P.S. we shall have $p_s = 2\pi \times 1500$. The wave length λ_s^1 on this corresponding uniform conductor for frequency of 1500 P.P.S. is very nearly

$$\lambda_s^1 = 33 \text{ miles.}$$

Hence, since the interposed coils are one mile apart we shall have

$$\sin \tfrac{1}{2} \varphi^1 = \sin \pi/33.$$

Now, $\sin \pi/33 = \pi/33$ to within one-sixth of one per cent. of the value of $\pi/33$. Up to this degree of approximation will, then, the non-uniform conductor just described have the same wave length and the same attenuation constant as its correspond-

ing uniform conductor. For lower frequencies the degree of approximation will be much higher; hence, for all frequencies which are of any importance in the telephonic transmission of speech the two conductors are equivalent.

B. Waves of Free Period.

Equation (4) holds for all values of D; it should therefore hold good when $D = O$. In this case, however, all the currents will vanish unless the denominator vanishes—that is, unless

$$\sin 2\,\psi \sin 2\,(k+1)\,\psi = 0 \ldots\ldots (4^a).$$

This transcendental equation, together with the equation

$$\frac{h}{\mu} \sin \frac{\mu\, l}{k+1} + 2 \cos \frac{\mu\, l}{k+1} = 2 \cos 2\,\psi \qquad (4b)$$

determines the free periods and their corresponding damping constants.

Equation (4a) will be fulfilled when

$$\psi = \frac{r\,\pi}{2\,(k+1)}$$

where r is any integer from 0 to ∞.

When $r = s\,(k+1)$ then we shall have not only

$$\sin 2\,(k+1)\,\psi = 0, \text{ but also } \sin 2\,\psi = 0.$$

These particular values of ψ give a set of free oscillations which form a distinct group, called further below the oscillations of *normal period*.

Equation (4b) should, therefore, be written:

$$\frac{h_r \sin \frac{\mu_r\, l}{k+1}}{\mu_r} + 2 \cos \frac{\mu_r\, l}{k+1} = 2 \cos \frac{r\,\pi}{k+1} \qquad (4c)$$

This equation will be referred to as the *period equation*.

It should be observed that

$$-\mu_r^2 = C\,(\nu_r^2\, L + \nu_r\, R), \quad h_r = C\,(\nu_r^2\, \lambda_0 + \nu_r\, R_0)$$

An inspection of (5) shows that the complete solution for waves of free period can now be written:

$$y_m = \sum_{r=1}^{\infty} \frac{A_r}{\sin \frac{r\pi}{k+1}} \left\{ \cos(k-m+2) \frac{r\pi}{k+1} \sin \mu_r \left(\frac{l}{k+1} - \xi \right) + \cos(k-m+1) \frac{r\pi}{k+1} \sin \mu_r \xi \right\} = \sum_{r=1}^{\infty} \eta_r$$

In the case of a uniform wire the periods and the wave lengths of the simple harmonic components $\eta_1\ \eta_2 \ldots \eta_r \ldots$ have in general a harmonic relation. The introduction of the reactance points $L_1\ L_2 \ldots$ disturbs this relation in consequence of a displacement in the values of the normal periods and of the normal damping constant. It is evident from purely physical considerations that since the introduction of the reactance points increases in general the inductance per unit length the periods will be lengthened.

8. Waves of Normal Period.

According to equation (7), Sec. I, the wave lengths of free oscillations on a uniform loop are as follows:

$$\frac{2l}{1}, \frac{2l}{2}, \frac{2l}{3}, \ldots \frac{2l}{k+1}, \frac{2l}{k+2}, \ldots \frac{2l}{2(k+1)}, \frac{2l}{2(k+3)},$$

$$\ldots \frac{2l}{s(k+1)}, \frac{2l}{s(k+1)+1} \ldots$$

Divide this series into groups of $(k+1)$ members each. Consider now the last member of each group; call it the *terminal member*. The wave lengths of the terminal members are

$$\frac{2l}{(k+1)}, \frac{2l}{2(k+1)}, \ldots \frac{2l}{s(k+1)}, \ldots$$

These terminal members will appear with normal periods among the free oscillations on the periodically loaded loop. To show this, consider the components of y_m of type $\eta_{s(k+1)}$, where

s may be any integer between 1 and ∞. Since in the case of these components $\phi = s\pi$ the period equation becomes:

$$\frac{h}{\mu} \sin \frac{\mu l}{k+1} + 2 \cos \frac{\mu l}{k+1} = 2 \cos s\pi.$$

This equation will be satisfied by putting

$$\mu = \frac{(k+1)\, s\pi}{l}.$$

The free periods of components of type $\eta_{s(k+1)}$ are now easily obtained.

$$\nu_s^2 L C + \nu_s R C = -\mu^2 = -\frac{(k+1)^2 s^2 \pi^2}{l^2}$$

$$\therefore \quad \nu_s = -\frac{R}{2L} \pm i \sqrt{\frac{1}{LC} \frac{(k+1)^2 s^2 \pi^2}{l^2} - \frac{R^2}{4L^2}}$$

$$= -\frac{R}{2L} \pm i p_s.$$

From the expressions for y_m we deduce:

$$\eta_{s(k+1)} = B_{s(k+1)} \sin \frac{(k+1)\, s\pi\xi}{l},$$

where

$$B_{s(k+1)} = B'_{s(k+1)}\, e^{-\frac{Rt}{2L}} \cos\left(p_{s(k+1)}\, t - \varepsilon_{s(k+1)}\right).$$

The wave length of this component is $2l/s(k+1)$.

The wave lengths of the components of type $\eta_{s(k+1)}$ are:

$$\frac{2l}{k+1}, \quad \frac{2l}{2(k+1)}, \quad \frac{2l}{3(k+1)}, \quad \cdots \quad \frac{2l}{s(k+1)} \quad \cdots$$

From these wave lengths and their corresponding periods we see that the oscillations of the type $\eta_{s(k+1)}$ are identical with the terminal members of the groups into which the free oscillations on a uniform loop were divided above. The physical reason why these periods have not been disturbed by the introduction

of reactance points is easily seen. It is because they have their nodes at the reactance points. The other k members of each group have been disturbed by the presence of the reactance points. The character of this disturbance will be now discussed.

b. Waves of Displaced Period.

From the expression for y_m we deduce, by putting $\xi = 0$,

$$x_m = \sum_{r=1}^{\infty} \frac{A_r \cos (k - m + 2) \frac{r\pi}{k+1} \sin \mu_r \frac{l}{k+1}}{\sin \frac{r\pi}{k+1}}$$

$$= \sum_{r=1}^{\infty} \xi_{m,r}$$

First, consider the wave lengths of these oscillations of type $\xi_{m,r}$. Begin with the fundamental

$$\xi_{m,1} = \frac{A_1 \cos (k - m + 2) \frac{\pi}{k+1} \sin \mu_1 \frac{l}{k+1}}{\sin \frac{\pi}{k+1}}$$

As m increases from 1 to $k+2$ the angle $(k - m + 2)\pi/k+1$ diminishes from π to 0. In Fig. 4 the line $L_1 L_1$ represents the length of the loop. L_{k+2} is the middle point of it. Points $1, 2, 3 \ldots k+2$ mark the position of the reactance points. At these points measure off abscissæ equal to $\xi_{1,1}\, \xi_{2,1}\, \xi_{3,1} \ldots$. The extremities of these abscissæ will be on the harmonic curve 1. This curve represents the fundamental oscillation of displaced period. Its wave length is $2\,l$, the same as the wave length of the fundamental oscillation before the introduction of the reactance points.

The next component is

$$\xi_{m,2} = \frac{A_2 \cos (k - m + 2) \frac{2\pi}{k+1} \sin \mu_2 \frac{l}{k+1}}{\sin \frac{2\pi}{k+1}}$$

As m increases from 1 to k + 2 the angle $(k - m + 2)\, 2\pi/k+1$ diminishes from 2π to zero. It will, therefore, have four maxima and four zero points on the loop. Its wave length is l. This component is represented by curve II in Fig. 4.

Take now the last component but one of the first group of free oscillations of displaced period. It is

$$\xi_{m,k} = \frac{A_k \cos (k - m + 2) \dfrac{k\pi}{k+1} \sin \dfrac{\mu_k l}{k+1}}{\sin \dfrac{k\pi}{k+1}}$$

As m increases from 1 to k + 2 the angle $(k - m + 2)\, k\pi/k+1$ diminishes from $k\pi$ to zero. This component has, therefore,

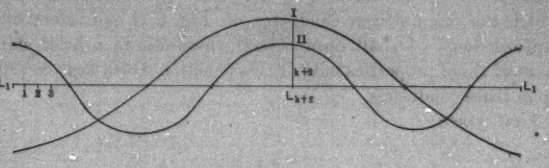


FIG. 4.

2 k maxima and 2 k zero points on the loop. Its average wave length is $2l/k$. The last component of this group is the same as the lowest oscillation of normal period—that is, γ_{k+1}.

The components of the first group have, therefore, the same average wave lengths as the components of the first group of free oscillations on the loop before the introduction of the reactance points.

It can now be easily shown that this is true for the components of all the other groups, and attention should be called here to the fact that the wave lengths of the free oscillations of displaced period can be considerably smaller than the distance between two consecutive reactance points. The introduction of the reactance points changes, therefore, neither the number of free oscillations nor their average wave length. Their periods, however, and their damping constants will be changed, as will be shown presently.

Displacement of Periods.

The periods and damping constants of the oscillations of displaced period are determined from the period equation

$$h_r \sin \frac{\mu_r l}{k+1} + 2 \cos \frac{\mu_r l}{k+1} = 2 \cos \frac{r\pi}{k+1}.$$

The unknown quantity ν_r contained in h_r and μ_r is, of course, a complex quantity—that is we can write

$$\nu_r = n_r + ip_r.$$

I am not as yet quite ready to discuss fully the general properties of the roots of the period equation. Besides, such a discussion would go considerably beyond the limits of this paper, the principal object of which is to determine the conditions under which the non-uniform conductor of Fig. 3 is equivalent to a uniform loop. I shall limit myself, therefore, to a brief statement of those properties only of this equation which bear directly upon these conditions.

First, since

$$\cos \frac{r\pi}{k+1} = \cos \frac{[2s(k+1) \pm r]\pi}{k+1}$$

it follows that the periods and the damping constants of the wave lengths

$$\frac{2l}{r}, \frac{2l}{2(k+1) \pm r}, \frac{2l}{4(k+1) \pm r}, \cdots \frac{2l}{2s(k+1) \pm r}, \cdots$$

are determined from one and the same angle $\phi = r\pi/(k+1)$. This relation between these wave lengths is illustrated graphically in Table I. The horizontal top bar marked 1, 2, ...k+1,.... 2k+1, 2k+2, 2k+3..... contains a numerical record of all the possible wave lengths

$$\frac{2l}{1}, \frac{2l}{2}, \cdots \frac{2l}{k+1}, \cdots \frac{2l}{2k+1}, \frac{2l}{2k+2}, \frac{2l}{2k+3} \cdots$$

The first vertical column marked 0, 1, 2, 3,. ... k + 1.....

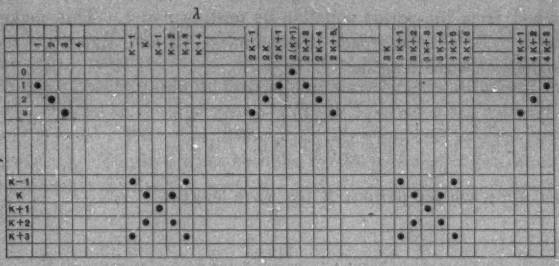
contains the record of all possible values of r. The meaning of the black dots is now easily explained. Take, for instance, the row of black dots in the horizontal bar 3. These dots are in columns 3, $2k-1$, $2k+5$, $4k+1$, $4k+7$, ... This means that if in the period equation

$$\frac{h}{\mu}\sin\frac{\mu l}{k+1}+2\cos\frac{\mu l}{k+1}=2\cos\frac{r\pi}{k+1},$$

we put $r=3$ then the roots of the equation will give us the periods and the damping constants for the wave lengths

$$\frac{2l}{3},\ \frac{2l}{2k-1},\ \frac{2l}{2k+5},\ \cdots\cdots$$

TABLE I.



For this reason these may be called the *concomitant wave lengths*. An inspection of this table shows that the concomitant wave lengths are symmetrically arranged with respect to the wave lengths of normal periods

$$\frac{2l}{k+1},\ \frac{2l}{2(k+1)},\ \frac{2l}{4(k+1)},\ \cdots\cdots$$

Of a given series of concomitant wave lengths there is one in each of the groups mentioned above. The first one of these groups contains the longest wave lengths; it may be called the *fundamental group*. The wave lengths of this group, together with their concomitant wave lengths, give all the possible wave lengths. If in the period equation r is given successively the

values 0, 1, 2, ... k + 1, the roots of the resulting k + 2 equations will give the periods and damping constants not only for the wave lengths of the fundamental group, but also their concomitants—that is, for all the wave lengths. It should be observed that the more nearly $\cos r\pi/(k+1)$ approaches the value +1 or —1 the more closely will the concomitant wave-lengths approach the wave lengths of normal period, around which they are symmetrically grouped, and therefore the more closely will their periods approach the period of the central wave length. For instance, the wave lengths

$$\frac{2l}{2k+1} \text{ and } \frac{2l}{2k+3}, \frac{2l}{4k+3} \text{ and } \frac{2l}{4k+5} \cdots\cdots$$

are concomitants of wave lengths $2l/1$. Their periods are obtained from the period equation by putting $r = 1$. This makes $\cos r\pi/(k+1)$ approach the value + 1, and therefore the periods of wave lengths

$$\frac{2l}{2k+1} \text{ and } \frac{2l}{2k+3}, \frac{2l}{4k+3} \text{ and } \frac{2l}{4k+5}, \cdots\cdots$$

will approach the periods of the wave lengths

$$\frac{2l}{2(k+1)}, \frac{2l}{4(k+1)}, \cdots\cdots$$

which have been calculated above. When, therefore, k is large then the roots of the period equation

$$\frac{h}{\mu}\sin\frac{\mu l}{k+1} + 2\cos\frac{\mu l}{k+1} = 2\cos\frac{r\pi}{k+1},$$

come out in pairs of nearly equal magnitude for all values of r, which are small in comparison to k. This peculiar property of the roots of the period equation suggests a striking resemblance between the oscillations of a periodically loaded conductor and the luminous vibrations of an incandescent gaseous substance, which, in my opinion, deserves serious attention. I expect to discuss this matter more fully at some future occasion.

EQUIVALENCE OF A NON-UNIFORM CONDUCTOR OF THE SECOND TYPE TO ITS CORRESPONDING UNIFORM CONDUCTOR.

Suppose that we now increase the number of reactance points without increasing the total reactance and resistance introduced into the line. In the expression

$$h = C(\nu^2 \rho + \nu R)$$

put

$$\rho = \frac{l \lambda^1}{k+1}, \quad R = \frac{l R^1}{k+1}$$

Hence,

$$h = \frac{l}{k+1} C(\nu^2 \lambda^1 + \nu R^1) = \frac{l}{k+1} h^1$$

As the number of reactance points is increased λ^1 and R^1 are to be kept constant; hence, ρ and R must vary inversely as $k+1$. When k becomes infinite then all the wave lengths different from zero will be in the first group of Table I between the columns 1 and $k+1$. The other columns of this table lose their physical meaning. To show what becomes of the period equation when k approaches the limit ∞ write the period equation in the following form:

$$\frac{\frac{l}{k+1} h_1 \sin \frac{\mu l}{k+1}}{\mu} - 4 \sin^2 \tfrac{1}{2} \frac{l \mu}{k+1} = -4 \sin^2 \tfrac{1}{2} \frac{r \pi}{k+1}$$

Since wave lengths different from zero are to be considered, finite values only of r need be considered. Hence, when $k = \infty$ the period equation becomes

$$l^2 (h^1 - \mu^2) = -r^2 \pi^2$$

or

$$C[\nu^2 \lambda^1 + \nu R^1 + \nu^2 L + \nu R] = -\frac{r^2 \pi^2}{l^2},$$

or

$$C[\nu^2 (\lambda^1 + L) + \nu (R^1 + R)] = -\frac{r^2 \pi^2}{l^2}.$$

This equation is, as it ought to be, the period equation of a uniform conductor of inductance $\lambda' + L$, resistance $R' + R$, capacity C per unit length. This conductor is the so-called corresponding uniform conductor of the periodically loaded loop.

When k is large but not infinite the same relation will exist approximately, and it is proposed to ascertain now the degree of this approximation.

We have seen that $l\,\mu\,/\,(k+1)$ varies with r as follows:

$$\frac{l\,\mu}{k+1} = s\,\pi, \quad \text{where } r = s\,(k+1),$$

s being any integer from 0 to ∞. Thus, when

$$\frac{r\,\pi}{k+1} \text{ is } 0,\ \pi,\ 2\,\pi,\ 3\,\pi,\ 4\,\pi, \ldots\ldots\ldots$$

$$\frac{\mu\,l}{k+1} \text{ will be } 0,\ \pi,\ 2\,\pi,\ 3\,\pi,\ 4\,\pi \ldots\ldots\ldots$$

Since $\mu\,l/(k+1)$ varies continuously with r it follows that when $r\,\pi\,/\,(k+1)$ is small $\mu\,l/(k+1)$ will also be small, since both vanish simultaneously. It is well to introduce here the physical meaning of $r/(k+1)$. Let λ_r be the wave length of the wave for which $\phi = \frac{1}{2}\,r\,\pi\,/(k+1)$, then

$$\lambda_r = \frac{2\,l}{r}, \quad \text{or } r = \frac{2\,l}{\lambda_r}$$

Therefore,

$$\frac{r\,\pi}{k+1} = \frac{2\,\pi}{\lambda_r}\,\frac{l}{k+1} = 2\,\pi\,\frac{l_1}{\lambda_r},$$

where l_1 is the distance between two consecutive coils. Let $2\,\pi$ be the angular distance of λ_r and θ_r be the angular distance of l_1 corresponding to the period of λ_r, then

$$\frac{l_1}{\lambda_r} = \frac{\theta_r}{2\,\pi},$$

Therefore,

$$\frac{r\,\pi}{k+1} = \theta_r.$$

Since for small values of $r\pi/(k+1)$ the angle $\mu l/(k+1)$ is nearly equal to $r\pi/(k+1)$ it follows that within these limits $\mu l/(k+1)$ is approximately equal to the angle θ_r. It follows, therefore, that wave lengths for which θ_r is sufficiently small the period equation

$$\frac{l}{k+1}\frac{h^1}{\mu}\sin\frac{\mu l}{k+1} - 4\sin^2\tfrac{1}{2}\frac{l\mu}{k+1} = -4\sin^2\tfrac{1}{2}\frac{r\mu}{k+1}$$

can be written

$$\left(\frac{l}{k+1}\right)h^1 - \left(\frac{l}{k+1}\right)^2\mu^2 = -\left(\frac{r\pi}{k+1}\right)^2,$$

or

$$C[\nu_r^2(\lambda^1+L)+\nu_r(R^1+R)] = -\left(\frac{r\pi}{k+1}\right)^2,$$

that is, the periodically loaded conductor has approximately the same period, damping constant and, of course, the same wave-length as its corresponding uniform conductor. The degree of approximation is of the same order as the degree of approximation between one-half of the angular distance separating two consecutive coils and the sine of that distance.

It should be observed that if the whole loop is divided into a number of equal parts and the consecutive parts are connected to each other by mutual induction, then the non-uniform conductor thus obtained will act in the same way as the non-uniform conductor of Fig. 3 of this section. Another arrangement, which is equivalent to the two arrangements so far described, is obtained by placing bridges at periodic intervals of the uniform loop, each bridge consisting of a coil of proper resistance and inductance, and the distance between the bridges being adjusted in accordance with the general rule formulated here. The mathematical analysis of these two arrangements does not differ essentially from the one given here, and can be easily worked out.

A rule governing the degree of approximation between a non-uniform conductor and its corresponding uniform conductor has thus been established, and it has been proved that the same rule is applicable to both types of non-uniform conductors, and also to both forced and free oscillations. The principal object of this investigation has, therefore, been accomplished.

American Institute of Electrical Engineers,

PARIS MEETING.

NEW YORK, June 4th, 1900.

DEAR SIR:

The following list has been compiled from the replies received to inquiries regarding the Paris meeting, the date of which has been fixed for August 16th.

MEMBERS WHO EXPECT TO ATTEND.

Foreign residents in *Italics*.

E. K. Adams,
D. Adamson,
Harry Alexander,
A. E. Appleyard,
J. H. Bowman,
Charles S. Bradley,
E. G. Bernard,
H. D. Betts,
M. Coster,
F. B. Crocker,
E. Egger,
George Forbes,
Charles Flesch de Nordwall,
S. B. Fortenbaugh,
E. T. Gilliland,
W. E. Goldsborough,
W. J. Hammer,
J. C. Hatzel,
R. O. Heinrich,
Carl Hering,
G. S. Hessenbruch,
W. L. Hildbergh,
N. M. Hopkins,
W. S. Hulse,
R. M. Hunter,
Samuel Insull,
A. E. Kennelly,
F. A. La Roche,
H. Ward Leonard,
R. W. Lohmann,
Alexander Macfarlane,
C. O. Mailloux,
L. B. Marks,
E. W. Mix,
H. L. Meyer,
R. D. McCarty,
H. D. McVay,
James Mitchell,
E. J. Molera,
W. E. Moore,
W. M. Mordey,
P. R. Moses,
C. H. Merz,
R. J. Nunn,
H. C. Parker,
C. P. Poole,
R. W. Pope,
Jesse M. Smith,
W. B. Potter,
P. H. Powell,
W. H. Preece,
M. I. Pupin,
E. Rathenau,
Calvin W. Rice,
A. E. Brooke-Ridley,
A. M. Schoen,
A. B. See,
E. A. Sperry,
George O. Squier,
T. Stahl,
John J. Swann,
F. Townsend,
H. Tripier,
VanRensselaer Lansingh,
W. D. Weaver,
E. G. Willyoung,
J. H. West,
G. H. Winslow,
Capt. E. L. Zalinski,

Total, 69.

WILL POSSIBLY ATTEND.

G. S. Albanese,	F. V. Henshaw,
William A. Anthony,	D. C. Jackson,
B. J. Arnold,	W. B Jackson,
Morgan Brooks,	Max Osterberg,
Douglass Burnett,	Andrew Pinkerton,
W. E. Case,	E. F. Price,
H. B. Coho,	F. Reichmann,
F. W. Darlington,	Harris J. Ryan,
S. E. Doane,	H. H. Wait,
H. G. Field,	E. R. Weeks.
C. J. Goldmark,	

Total, 21.

CONVERSAZIONE IN LONDON, JULY 5TH, 1900.

THE INSTITUTION OF CIVIL ENGINEERS,
Great George St., Westminster, London.

January 26th, 1900.

RALPH W. POPE, ESQ., Secretary,
AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS,
26 Cortlandt Street, New York, N. Y.

DEAR SIR:

It is my pleasure to acknowledge the receipt of your letter of the 16th instant, and to say that Sir Douglas Fox (the President) is very glad, indeed, to learn that he may possibly have the pleasure of welcoming a number of your members in London on their way to Paris.

May I take this opportunity of saying that it has just been arranged to hold the Annual Conversazione of the Institution on Thursday, the 5th July, when the President and Council hope that the large number of members of this Institution from all parts of the country, who are drawn annually to that function, may have an opportunity of meeting many of their American professional brethren. A formal invitation in respect of your visit to this country will presently be sent, when its date is more fully determined on by you; but I meanwhile mention this fixture, as it has just been made.

Hoping to hear further from you after the Council meeting to which you refer,

Believe me, yours faithfully

(Signed) J. H. T. TUDSBERY, Secretary.

THE INSTITUTION OF CIVIL ENGINEERS.

PRESIDENT'S RECEPTION.

Writing under date of May 9th, Secretary Tudsbery says:

"In respect to the visit of members of your Society in July, which is anticipated here with so much pleasure, I am desired by the President to inform you, and request you to make it known to the members of the American Institute of Electrical Engineers, that he proposes to formally welcome the visitors in the Institution Library, at 3 o'clock, on the afternoon of Monday, the 2d of July. Thereafter, during that week, the accommodation of the Institution rooms will be entirely at the disposal of your duly accredited members; and certain fixtures of a social kind are now under consideration by our Reception Committee, to attend which invitations will be sent to the visitors, at their addresses on this side, which they should intimate to me as soon after their arrival as convenient to them, if they cannot do so previously.

With regards to details of the arrangements referred to, I hope to be in a position to inform you further presently."

The Secretary will sail from New York August 1st, on the White Star Steamship *Majestic*. Those who have arranged their sailing dates, are requested to

send them, also their London and Paris addresses, in order that communications may be forwarded if necessary.

The following cablegram has been received:

PARIS, May 17th, 1900.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS, in Convention at Philadelphia:

Have furnished Institute permanent headquarters in Palace Electricity.

DRAKE, Director.

THE PARIS MEETING.

THURSDAY, AUGUST 16th, 1900.

The proposed programme for the Paris meeting in August next will embrace various attractive social features. Mr. T. C. Martin, of the Committee, cables from London that the Institution of Electrical Engineers will offer a dinner in London to the visiting members of the INSTITUTE on August 13th. The members of both bodies will take a special train to Paris on August 15th. The joint meeting will be held in the United States building at the Paris Exhibition on August 16th and on the evening of the same day a reception will be held at the British building, the headquarters of the British Royal Commission.

The joint meeting proper on August 16th, will be limited to a single session of three hours, and probably no papers will be read.

Yours truly,

RALPH W. POPE, Secretary.

26 Cortlandt St., New York, June 4th, 1900.

PATENT EXPERT.

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Consulting Electrical Engineer.
31 Nassau St., New York.
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